Dynamical Relativity and Algebraic Dynamicism

Abstract. *Dynamicism* is the view that the dynamical features are more fundamental than the geometrical features of the world. In the context of general relativity, the chronogeometrical significance of the metric field—considered as a matter field—is derived from the dynamical laws. The opposite view that geometry is fundamental and presumed by dynamics is called *geometricism*. In this paper, I renew the support for dynamicism with the reason that positing (chrono)geometrical principles as fundamental leads to more complicated ontology and ideology, and imposes unwarranted restrictions on the behaviors of matter. I also revisit and respond to two objections to dynamicism by Norton (2008). Following Menon (2019), I adopt *algebraicism* (a framework that does not posit a manifold) in response, but I argue that Menon's proposal based on a traditional formalism of algebraicism is seriously flawed. Instead, I suggest a solution—which I call *dynamical algebraicism*—based on the new formalism by Chen and Fritz (2021).

Keywords. Physical relativity; dynamicism; geometricism; algebraicism; Einstein algebras; field algebras; natural operators.

1 Introduction: Geometricism vs Dynamicism

Does space (or spacetime) exist? This question is the central focus of the traditional debate between *absolutism* and *relationalism*. Absolutism says that space exists as the container of matter and material processes, while relationalism says that it is reducible to relations between material bodies. The modern development of relativistic theories seems to adjudicate in favor of absolutism. According to the standard interpretation of general relativity, spacetime not only exists but has a curvature, which can be formulated as a metric field and is determined by the distribution of mass-energy (for example, see Friedman 1983).

In light of this, the old debate is partly transformed into a new one between *geometricism* and *dynamicism*, focusing on the ideological status of (chrono)geometrical notions and laws. According to geometricism, the geometrical features of our world are fundamental, exist independently from dynamical laws, and are presumed by the latter. According to dynamicism, it is opposite: the dynamical laws are fundamental and the geometrical features and laws are derived from them. In the context of general relativity, geometricists hold that the metric field represents the fundamental geometrical feature of spacetime, while dynamicists hold that the (chrono)geometrical significance of the metric field (as a matter field) is derived from the dynamics of measuring devices like rods and clocks that survey the field. Note that in the traditional debate between absolutism and relativism, both sides typically assume geometricism: for example, both Newton and Leibniz considered the notion of spatial distances to be fundamental. Maudlin (1993) argues for the relational interpretation of special relativity, which also posits geometric notions as fundamental.

Geometricism has been the more standard position in the literature, advocated by authors like Friedman (1983), Janssen (2002) and Maudlin (2012). Like absolutists, the geometricists often appeal to the explanatory role played by the primitive geometrical feature of spacetime. For example, clocks behave similarly despite their different matter constitutions *because* they track the same spacetime intervals. On the other hand, Brown (2005) has famously argued for dynamicism, denying the explanatory power of the spacetime geometry. As such, the debate is often construed as being strictly about the *explanatory order* between geometry and dynamics (see Janssen 2002, Brown 2005, Brown and Read 2021). While explanatory order is central to the debate, merely focusing what explains what and what needs to be explained can lead to a stalemate and is criticized by many authors including Norton (2008), Martínez (2007) and Knox (2017). Therefore, I construe the debate as primarily about comparing two approaches to spacetime theories that differ in the ideological status of geometry and dynamics. I aim to renew the support for dynamicism by arguing that holding (chrono)geometrical laws to be fundamental not only leads to a more complex ontology and ideology but also imposes unwarranted restrictions on the behaviors of matter (Section 2).

Should the debate have ontological import, namely on the ontology of spacetime?[1] Authors like Janssen (2002) and Brown (2005) think that the ontological question is orthogonal to the debate. Others such as Norton (2008) and Acuña (2016) argue that the ontology should be relevant and even the primary focus of the debate. I agree with Acuña that geometricists ought to postulate substantival spacetime in order to claim the explanatory virtue of their approach, and that dynamicists ought to deal away with substantival spacetime to reap the full benefits of dynamicism. Therefore I do not treat ontological claims as completely orthogonal to the debate and will rather focus on evaluating *substantival geometricism* (there is spacetime with geometrical features at the fundamental level) and *relational dynamicism* (there is neither spacetime geometry nor substantival spacetime at the fundamental level; call the dynamical view that posits spacetime *substantival dynamicism*). For example, the basic chronogeometrical laws under geometricism that I will posit assume the existence of spacetime.

[1]Here, "spacetime" is construed broadly. Some people *define* spacetime as a manifold equipped with a geometry determined by a metric field. It automatically follows from this definition that dynamicism (which does not posit fundamental geometric features) rejects spacetime at the fundamental level. But broadly construed, spacetime is just an arena where physical fields live on and physical processes happen. It can be a bare manifold with only a topological structure.

After motivating relational dynamicism, I will defend the view from Norton's (2008) two objections (Section 3). I reject one objection that appeals to the explanatory power of fundamental spacetime, but acknowledge the other objection as important. Roughly, Norton argues that dynamicists must presume at least the notion of spacetime coincidence, which is a geometrical notion, and thus are mistaken in holding that geometry is entirely derived from dynamics. To adequately address this objection, we need a technically feasible framework that deals away with manifolds or an underlying arena that physical fields live on.

A good candidate for such a framework is *algebraicism*. A core feature of an algebraic approach is that it does not posit manifolds as an underlying arena for functions and fields to live on. For example, in the standard formalism proposed by Geroch (1972), all information about a manifold is encoded in the algebraic structure consisting of all smooth functions on the manifold. These functions are no longer analyzed as maps from the manifold to real numbers, but are reconceptualized as simple elements in the algebraic structure defined by all the algebraic operations we can perform on them. Geroch shows that we can do physics with them up to general relativity. Menon (2019) proposes a response to Norton based on this formalism, pointing out that this can help make sense of spacetime coincidence without assuming spacetime points (since there is no manifold). However, I think this particular marriage between dynamicism and algebraicism is a mistake, because his algebraic formalism does not sit well with relational dynamicism (Section 4.1). For one thing, the most natural interpretation of the basic algebraic structure posited there is spacetime. Indeed the position is sometimes called "algebraic substantivalism" (see Earman 1989, Bain 2003; also mathematicians sometimes call the algebraic structure "spacetime"). It does not correspond to any matter field recognized by fundamental physics and carries no energy and momentum. It is just another representation of (metrically amorphous) spacetime traditionally represented by a manifold.

Instead, I introduce a new algebraic response to Norton's argument based on Chen and Fritz's (2021) formalism of *field algebras*, which I call *dynamical algebraicism* (Section 4.2). According to this approach, all physical fields are treated on a par, and the basic algebraic structure consists of only physical fields that are acknowledged by current physics (which can be expanded if more are discovered). The technical challenge this approach faces is to codify different types of physical fields (for example, a one-form field, a tensor field of rank (0,2), a spinor field) within a single algebraic structure, unlike Geroch's formalism where all elements of the basic algebraic structure are smooth functions. This challenge can be met by using the technique from category theory (a framework known for its power of generality). Roughly, a field is first conceptualized as a *functor* from the category of manifolds to the category of sets that assign field configurations to manifolds. Then, fields of different types are reconceptualized as elements of an algebraic structure characterized by *natural operators* on field functors. I argue that the dynamical approach based on this formalism can meet the challenges better than the other approaches.

Let me briefly remark on the relevance of this debate between geometricism and dynamicism for other contemporary issues in philosophy of physics. Even though general relativity is not a final theory of spacetime due to its conflict with quantum theory, the discussion on the interpretation of relativistic theories is not obsolete. Precisely because there are lots of obscurities around how to develop quantum gravity (a research program unifying general relativity and quantum theories), thinking more clearly about relativity can be important (after all, we can use some help from all angles). Moreover, exploring manifold-free frameworks provides more possibilities for formulating quantum gravity.[2]

[2]For instance, the formalism of noncommutative geometry in the algebraic framework cannot be easily interpreted in terms of manifold points (see Huggett et al. 2021). Dynamicism and quantum physics are also connected in other ways. It might be worth mentioning that I argue elsewhere that we can use quantum physics to argue for dynamicism in discrete spacetime, which is a particularly conceptually clear case for dynamicism: the apparent geometry emerges from the dynamical laws defined on discrete space devoid of geometrical structures ([Anonymized]).

2 The Case for Dynamicism

I will argue that relational dynamicism, the view that spacetime and its geometry is less fundamental than dynamical laws, is better than substantival geometricism, which says otherwise, after laying out the basic principles under the geometrical and dynamical interpretations of special and general relativity.

2.1 Geometricism

Substantival geometricism (henceforth simply "geometricism") says spacetime exists independently of matter, and has an intrinsic geometry that plays a role in determining how matter behaves. Norton (2008), who defends this view (which he calls "realism"), puts it into three claims in the context of special relativity (STR). The first two claims are as follows:

> GEOMETRICISM-1$_{STR}$. There exists a spacetime that can be coordinated by a set of standard coordinates (x, y, z, t) that are related to each other by Lorentz transformation.
>
> GEOMETRICISM-2-$_{STR}$. The spatiotemporal interval I between any two spacetime points (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) is an intrinsic property of spacetime independent from processes it contains, and is given by $I^2 = (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2$.

If $I^2 > 0$, then I is time-like, which corresponds to the time elapsed on an ideal clock traveling straight from one point to the other, and if $I^2 < 0$, then I is space-like, which corresponds to the distance measured by an ideal rod.

Let me spell out the general relativity (GTR) version of the two claims:

> GEOMETRICISM-1$_{GTR}$. There exists a spacetime that can be coordinated by a set of standard coordinates (x, y, z, t) that are related to each other by general coordinate transformations.

Geometricism-2$_{GTR}$. Metric is an intrinsic property of spacetime independent of the process it contains, and is (partially) determined by the distribution of mass-energy in a way specified by the Einstein field equations. The spatiotemporal intervals surveyed by ideal clocks and rods are determined by the metric.

The third claim Norton proposed should apply to both STR and GTR:

Geometricism-3. Material clocks and rods measure spatiotemporal intervals because there are chronogeometric-dynamical laws that specify the relation between the geometry of spacetime and the behaviors of matters.

What are those chronogeometric-dynamical laws? I formulate them as follows (following Maudlin 2012):[3]

The Null Hypothesis. The trajectory of light is a null geodesic regardless of the physical state of its source.

The Geodesic Principle. The trajectory of any free system is a timelike geodesic.

The Clock Hypothesis. The amount of time recorded by an ideal clock is (proportional to) the length of its worldline.

The last principle may seem strange because "clock" does not seem a perfectly natural notion. (As Maudlin puts it: "Nature may recognize a distinction between light and

[3]One may object that geometricists only endorse these principles strictly within the context of special relativity. In that case, it is unclear what basic principles geometricists should endorse. A sensible option is for geometricists to endorse *exactly the same* mathematical equations as basic laws as dynamicists but apply slightly different metaphysical interpretations (e.g., calling the metric field a geometric structure while dynamicists call it a matter field). But in this case, I contend the debate would not be substantive (apart from the ontological disagreement between absolutism and relationalism).

I should add that positing these principles for both special and general relativity is not unmotivated: they give a unified framework for understanding spacetime theories (Newtonian mechanics, special and general relativity), which is a theoretic virtue.

massive particles, but Nature does not have to settle whether a given mechanism counts as a 'clock' in order to determine how it should behave." (Maudlin 2012, 106)) But a clock is just a system the state of which changes periodically, and when non-accelerating, each period corresponds to an equal spacetime interval. Of course, we cannot test whether an actual system behaves like an ideal clock by directly measuring how long each period of a clock lasts (which we have no access to without circularity). But we can compare the periods of different clocks. If their periodicities are all consistent relative to each other, then they can be considered accurate, because the best explanation for their synchronization is that they all correctly track spacetime intervals.

Luckily, clocks do tend to synchronize regardless of their particular constitutions.[4] This leads to a much-acclaimed explanatory power of the geometric interpretation. If there is no such thing as time or time-like intervals, then it seems to be a mysterious coincidence that clocks of all material constitutions tend to synchronize. But there is no such mystery: they synchronize simply because they track the metric of spacetime in accordance with the clock hypothesis. This also applies to other behaviors of matter: for example, why fast-moving rods appear shorter regardless of their particular matter constitutions—again, the reason is that they are correctly tracking the spacetime geometry responsible for the phenomenon of length contraction.

The geometric interpretation of general relativity is also praised for its explanatory power including dissolving the mystery of why different bodies fall at the same rate. It seems a mysterious coincidence that the gravitational force received by a body is proportional to its inertial mass so that the effect of mass happens to be canceled out when we calculate the gravitational acceleration. But this coincidence can be

[4]Of course, no actual clock is perfectly ideal. In practice, we use atomic clocks as the standard measurement of time. It is the successor to the standard of time called ephemeris time, which is based on Earth's orbit around the sun. Ephemeris time was abandoned due to random fluctuations of earth that make this standard unreliable. A well-built caesium atomic clock still has the risk of abnormality, so the current standard of time is actually based on the average of many such clocks in different laboratories (see Brown 2005).

explained away by geometricism. Since a free-falling body receives no force, its state is equivalent to that of floating in empty space. So we can understand different bodies falling to the ground equivalently as the ground is raised to meet them—of course the ground meets them at the same rate.

2.2 Dynamicism

Relational dynamicism (henceforth "dynamicism") is the view that spacetime and its geometry are not fundamental and can be derived from the dynamical laws governing matter. In the case of special relativity, the fundamental principle is simply that the dynamical laws are all *Lorentz invariant*. This means that the dynamical laws have the same form in coordinate systems related to each other by Lorentz transformation. To highlight:

> Dynamicism$_{STR}$. All physical laws are Lorentz invariant.

This principle is sufficient to ground all empirical implications of special relativity. To see it, we can recover the chronogeometric-dynamical laws from this one—not as pure postulates about physical reality but results of our construction (indeed, Norton calls dynamicism "constructivism").

First, the null hypothesis can be derived from the principle that the speed of light (or Maxwell's equations) is Lorentz invariant (under Einstein's synchronization convention), because this principle allows us to *construct* a Minkowski spacetime in which the trajectory of light is a null geodesic.[5] So, the null hypothesis is essentially reduced to the Lorentz invariance of speed of light, which is just an instance of Dynamicism$_{STR}$ applying to electromagnetism. The geometrical notions such as

[5]The Lorentz invariance of Maxwell's equations entails that the "two-way" speed of light (the speed measured in the trip from the source to the detector and back to the source) is Lorentz invariant. Einstein's synchronization convention stipulates that the one-way speed of light equal the two-way speed. Since the null hypothesis is not fundamental under dynamicism, we do not need this convention to be objectively true. Thanks for an anonymous referee at BJPS to point it out.

"Minkowski metric" and "null geodesics" are our constructions and do not correspond to the fundamental reality.

Similarly, the geodesic principle is reduced to that the physical laws in various matter theories are Lorentz invariant. Again, this permits us to construct a Minkowski spacetime with its timelike geodesics being the trajectories of free-falling material bodies. There are no antecedently existent geodesics that coincide with such trajectories. The clock hypothesis amounts to that the laws governing the matter that constitutes an ideal clock are Lorentz invariant (in addition to the conditions for being a clock[6]). We can recover the principle by defining the spatiotemporal interval of the worldline of an ideal clock to be the time it records.[7]

In the case of general relativity, the metric field is indispensable, and dynamicists cannot explain it away through the symmetries of other matter fields as in special relativity.[8] Thus, like geometricism, dynamicism needs to include the Einstein field equations as basic postulates of general relativity. But unlike geometricism, the metric field is treated as one of the matter fields and its chronogeometrical significance is considered as derived from the its interactions with other matter fields.[9] Since

[6]For example, the matter of a clock needs to have a time-like worldline in order to record proper time (in the geometrical terms).

[7]We need to appeal to clocks that function normally not only when moving inertially but also when accelerating (this is also true for geometricism). That is, when a clock is accelerating, we need to make sure that its periodicity is not affected by its acceleration (such disturbances, if exist, can be discovered in the same way as we discover the abnormality of an atomic clock or ephemeris time). This boils down to the requirement that the restorative force of a clock is stronger than the acceleration force. As Brown (2005) points out, this is analogous to the dynamical understanding of length in Euclidean space. According to dynamicism, the length of an object is defined by the behaviors of measuring devices such as rigid rods or strings. Whether such measuring devices still track length when they are bent depends on whether bending distorts the relevant properties of their constituents such as the comparative lengths of their atomic links.

[8]Of course, there are alternative equivalent formulations of general relativity that do not involve a metric field (see Krasnov 2020). But in those formulations, some other manifold-theoretic structures replace the metric field, such as differential forms, connections, or spins. It does not seem that switching to these structures will affect our discussions in important ways—we still face the same interpretative problems.

[9]Note that if we interpret the metric field as the geometrical property of other physical fields rather than of spacetime—which is analogous with assigning spacetime intervals as relations between events or material bodies in special relativity—this would not be a dynamicist view even if we do not posit spacetime. Again, the debate between geometricism and dynamicism is primarily about the ideological priority between geometrical and dynamical notions and laws.

the notion of "metric" has the geometrical connotation that it is a mathematical device for representing geometrical properties of spacetime, I will call the field *the gravitational field* in the context of dynamicism. Similarly to GEOMETRCISM-2_{GTR}, the fundamental principles under dynamicism includes:

> DYNAMICISM$_{GTR}$. The gravitational field is coupled with the distribution of mass-energy in accordance with the Einstein field equations.

It is not clear what other basic principles should be postulated in the dynamical interpretation of general relativity. As we will see in the upcoming discussion, the chronogeometric-dynamical laws cannot be straightforwardly recovered from DYNAMICISM$_{GTR}$ (although they can *nearly* be recovered).[10] However, I will argue that this is an advantage of dynamicism rather than a problem.

2.3 The case for dynamicism

I will argue that the dynamical interpretation of relativistic theories is better than the geometrical interpretation. In particular, I will argue that unlike geometricism, dynamicism does not impose unnecessary constraints on matter, allows a simpler ontology and ideology, and avoids some empirical underdetermination that troubles geometricism. Finally, I will argue that the case beyond relativity further bolsters the case for dynamicism.

In the literature, Brown (2005) famously argues for dynamicism, claiming that the genuine explanatory order runs from the dynamical laws to chronogeometrical laws. For example, he compares the explanatory power of spacetime and its geometry to the role played by "the geometries of the configuration space in classical mechanics" and "the space of equilibrium states in thermodynamics" (139), which presumably do

[10]It might be worth emphasizing again that we are not aiming to recover the principle as literally true, since there is no spacetime according to dynamicism, but only as to the effect that we can construct a spacetime that has the trajectories of free falling systems as its time-like geodesics. The same applies to other chronogeometrical laws.

not play an explanatory role, and thereby raises the question "why should spacetime geometry be any different?" (139) He also argues that "the conspiracy of inertia" (141) is a postulate without no explanation, and "anyone who is not amazed by this conspiracy has not understood it" (15). He goes on to claim that, with dynamicism, inertial motion is finally not "a miracle" (163) However, geometricists hold the geodesic principle as fundamental and in no need of further explanation. For example, Martinez (2007) rejects Brown's argument: "inertial motions are distinct, having different speeds and directions; how much more different would they have to be to not constitute a conspiracy?" (211)

Such a discussion, as many have pointed out, is not very productive (see Knox 2017, Norton 2008). Thus, I think we should avoid appealing to intuitive order of explanations and compare the overall merits of the resulting theories. I will argue that (substantival) geometricism not only postulates a more complicated ontology and ideology than (relational) dynamicism, which leads to more empirical underdetermination, but also imposes unnecessary constraints on the behaviors of matter.

Simplicity Comparing to geometricism, dynamicism posits fewer kinds of entities and fundamental structures of the world. In particular, dynamicism posits fewer kinds of entities because it does not posit substantival spacetime, which is a different kind of object from matter.[11] Dynamicism posits fewer fundamental structures because it does not posit chronogeometrical structures as basic, such as spatiotemporal intervals featured by geometricism.

While there is often a trade-off between ontological parsimony and ideological parsimony, it is not clear at all that dynamicism needs to posit more axioms than geometricism, if not the opposite. For example, in special relativity, dynamicism only has one basic principle constraining all dynamical laws, while geometricism has a lot

[11]The number of entities are similar, since for dynamicism, the gravitational field is a fundamental entity (in the context of relativity) in the place of spacetime for geometricism. But the gravitational field is not a different kind from other matter fields.

more. One may object that this is not a matter of simple counting—perhaps the second-order form of Dynamicism$_{STR}$ is inherently more complex despite it being one principle. But the second-orderness is not essential here, since the principle can just as well be about the coupling of the gravitational field and other matter fields, as shown in Dynamicism$_{GTR}$.[12]

Empirical underdetermination When one theory has more fundamental structure than another without empirical difference, it can be expected that the former would be inflicted with more problems of empirical underdetermination. I will consider two famous arguments against realism about spacetime and geometry.

The Poincare disk. The argument from the Poincare disk says that if there is a fundamental geometry, the citizens of a peculiar two-dimensional disk space cannot distinguish between two possibilities with any empirical measurements: a hyperbolic geometry with no universal force and a Euclidean geometry with a universal force that distorts measurement devices. As the moral of the story goes, the geometry is radically underdetermined by measurements—distinct geometries can be made consistent with measurements by choosing suitable dynamical laws. Assuming a radical underdetermination of this kind is problematic, it would trouble geometricism since the latter assumes there is an independent metric tracked by rigid rods in accordance with some chronogeometric-dynamic laws. The same argument does not apply to dynamicism, since the latter holds that empirical results are explained by dynamical laws alone. According to dynamicism, there is no separate metric field that determines the real geometry and a universal force that distorts the apparent geometry, but one and the same field and its dynamics responsible for the apparent geometry.

The hole argument. A hole transformation is a diffeomorphism on the spacetime

[12] I would also like to make a connection between dynamicism and fundamental "invariance groups" entertained by structuralists (for example, see McKenzie 2014, French and Ladyman 2003). The invariance groups consist of symmetries of physical fields, so the approach amounts to taking the symmetries of physical fields as fundamental, which is very much in the spirit of dynamicism.

manifold that is the identity map outside a region (called the "hole") but not identity inside it (see Earman and Norton 1987). According to the principle of general covariance, the world would be empirically indistinguishable before and after a hole transformation because the spreading of the metric field and of other matter fields over manifold points are transformed uniformly. Moreover, the world would be radically indeterministic because fixing all physical states outside the hole does not fix the physical states inside it, no matter how small the hole is. This is a problem because we do not want determinism or indeterminism become a trivial consequence of a metaphysics. This argument is meant to oppose *manifold substantivalism*, the view that manifold points represent spacetime points which are real individuals. Insofar as this view is a natural position for geometricists, the hole argument is a challenge to geometricism. The hole argument has a vast literature, and many geometricists have proposed solutions to it. For example, some argue that spacetime points do not have primitive identity (e.g., see Hoefer 1996, Pooley 2006).

Dynamicists can answer the argument by denying the reality of manifold and spacetime points (see Brown 2005, 156). According to this line, the hole transformation is only a transformation of coordinate systems—that is, a mere renaming of the relative positions of the metric field and other matter fields—and therefore does not any reflect change in the objective reality. Thus, there are no distinct states to be distinguished or to be determined. While the geometricists have viable responses to the hole argument, being able to give a new and seemingly cleaner solution still scores some points for dynamicism.

Unwarranted restrictions The null hypothesis says that light ray travels along null geodesics in vacuum. Some authors such as Menon et al (2020) have pointed out that the null hypothesis is not derivable from the Einstein-Maxwell equations (relativistic electromagnetism). As we known, light is an electromagnetic wave, and the ray-like trajectory of light is not always well-defined. For example, in optics, *Poisson spot* is the famous phenomenon that when light passes around a circular opaque object to a screen, there will be a bright spot on the screen in the middle of the shadow. But light can obtain a ray characterization through geometrical optics approximation, which holds when the wavelength of the light in question is significantly smaller than (1) the length scale on which its amplitude varies, and (2) the scale on which the curvature of spacetime is nonnegligible. The approximation can fail precisely in cases like Poisson spot. The null hypothesis can be obtained as the limit of geometrical optics, where the wavelength of the light tends to zero while its amplitude remains constant. But as Menon et al point out, this derivation is based on the assumption that the solutions to the Maxwell equations arrived at in this limit do approximate exact solutions to arbitrary accuracy. This assumption is nontrivial: it can be violated in some physically and epistemically relevant scenarios.

There is an extensive literature on the derivation of the geodesic principle from the Einstein field equations (e.g., Misner, Thorne and Wheeler 1973), in which it is often claimed that the geodesic behavior of a free falling system is a result of the vanishing covariant divergence of its mass-energy tensor, which in turn follows from the Einstein field equations (see Brown 2005, Malamant 2012, Weatherall 2011, Menon et al 2020). It is usually considered an achievement of general relativity to prove the principle rather than postulating it. However, various authors such as Tamir (2012) and Weatherall (2011) have pointed out that this derivation actually relies on additional assumptions. According to Tamir, we can only reasonably derive that free falling systems tend to cluster around time-like geodesics without achieving exact "geodicity."

The derivation of the clock hypothesis relies on the validity of local Lorentz invariance. Recall that in special relativity, this principle is derived from Lorentz invariance of physical laws that govern the matter constitutions of clocks. If Lorentz invariance holds of small areas where gravitational tidal effect is small, then we can recover the principle locally. And if clock surveys time-like intervals of its worldline locally, then adding them together gives the length of its worldline, which recovers the principle globally. However, local Lorentz invariance is not a consequence of the field equations. If we interpret "local" as infinitesimally small, then this amounts to the requirement that the gravitational field has Lorentz signature $(-+++)$. (In geometricism, this requirement is built into GEOMETRICISM-2_{STR}, namely how the spatiotemporal intervals are locally determined by the metric.) While the Einstein field equations typically apply to Lorentzian metrics, they can also apply to other metrics (see Dray et al 1997 for a variational metric as a solution to the field equations). Thus, the clock hypothesis is necessarily true in the context of general relativity.

According to these results, the chronogeometric-dynamical laws postulated by geometricism are only approximately true, which is a tell sign that they are derived from more fundamental principles. Positing them as basic principles that constrain the behavior of matter is unwarranted because these additional constraints do not account for the empirical success of general relativity and may be falsified by further empirical data. In contrast, dynamicism only posits the Einstein field equations (and principles of matter theories), and therefore does not impose these unnecessary constraints on the behavior of matter.

The pathway to quantum gravity Finally, I would like to comment on how dynamicism fares better than geometricism when we go beyond general relativity, in particular, concerning quantum theories. The intuition behind attributing basic chronogeometrical significance to the gravitational field has a lot to do with the special status of the field, namely that all matter fields are coupled with it. However, when we consider the programs of quantum gravity, the gravitational field behaves just like other matter fields. It not only carries energy and momentum but also subject to quantum fluctuations.[13] Although we do not yet have a fully formulated theory of quantum gravity as a candidate fundamental theory, there is however a fully formulated and mathematically consistent "effective" theory of gravity which holds at physical scales that are significantly larger than the Planck scale (but can still be smaller than any scale that we can probe with our current technology). The prescription for formulating this effective theory of gravity is exactly the same as formulating other effective theories of particle fields. For example, in this framework, a "graviton" is an excitation of the gravitational field (perhaps we should call it the graviton field now) in exactly the same formal sense as a photon being an excitation of the electromagnetic field in quantum electrodynamics. (For more details of the effective theory of gravity, see for example Burgess 2003, Crowther 2013.)

Thus it seems that we have good reasons to consider the gravitational field on a par with other matter fields. The geometricists then face a dilemma. It is implausible to insist that the gravitational field is just a mathematical device for encoding the chronogeometrical properties of spacetime.[14] But if they posit that the field is a real physical entity that lives on spacetime and is governed by dynamical laws (or effective dynamical laws in aforementioned effective theory) just like other matter fields, then it would be odd to consider it also to have the fundamental chronogeometrical significance on top of those dynamical laws.

One may object that many approaches to quantum gravity struggle with the problem of background dependence: they have to posit a fixed background spacetime (i.e., not subject to quantum effect) in addition to the quantized graviton field (e.g.,

[13]It is widely accepted that the metric field is subject to quantum effect (e.g., Eppley and Hannah 1977). Semi-classical gravity with a classical metric field is treated as only an approximation to the fundamental theory, although there are some skeptical voices (e.g., Mattingly 2001).

[14]It is worth mentioning that although many programs of quantum gravity are labeled as quantum geometry, they are not really committed to geometricism about the gravitational field. For example, Rovelli and Vidotto (2015) argue that spacetime geometry is discrete and fuzzy. Such a statement merely rests on the common assumption that the gravitational field permits a geometrical interpretation. Indeed, Rovelli seems to be against geometricism (Rovelli 1997, see also Brown 2005).

see Crowther 2013). But this is not a vindication of geometricism. On the contrary, since background dependence is seen as a clear problem to be solved, dynamicism plays a good methodological role in putting extra pressure on solving this problem.

3 Revisiting Two Objections to Dynamicism

Norton (2008) raises what I consider to be two important and perhaps representative objections to dynamicism. In essence, he argues that dynamicism leaves the coincidence among different matter theories unexplained, and that dynamicism tacitly assumes spacetime as a fundamental entity. The first objection applies to both substantival dynamicism and relational dynamicism, and the second only targets at the latter. I reject the first objection as weak but consider the second one important. To fully respond to the second objection, I contend that we need to go beyond the traditional framework of differential geometry as well as other existent proposals (in particular, Menon's (2019) proposal that I will discuss in the next section).

First, Norton argues that dynamicism has to either (1) leave a coincidence among different matter theories unexplained, or else (2) commit to an extreme form of operationalism. For example, clocks tend to synchronize regardless of their particular constitutions, which are studied by different matter theories. If there is no such thing as time or time-like intervals, this would seem to be a mysterious coincidence. But there is no such mystery: they synchronize simply because they track the metric of spacetime in accordance with the clock hypothesis. Similarly, why do fast moving rods appear shorter regardless of their particular matter constitutions? Again, this is not a mysterious coincidence and the reason is that they are correctly tracking the spacetime geometry responsible for the phenomenon of length contraction. Norton compares this explanatory task to "footprints on a sandbeach": if we see many identical footprints on a sandbeach, we will infer that there was a foot that produced them. Thus we should infer that spacetime exists and that rods and clocks measure the metric of spacetime. But then, one has to accept a spatiotemporal structure existing prior to matter, unless one embraces extreme operationalism, according to which quantities like temporal durations do not exist until they are measured.

Since the second horn of the dilemma is indeed implausible, the core of the argument lies in the first horn. In special relativity, does the fact that all physical laws are Lorentz invariant imply that there is a Minkowski spacetime? Not necessarily. The principle of common cause that coincidences should be explained by a common cause is really grounded in that the resulting theory that posit the common cause is simpler or has other theoretical virtues—for if this were a fundamental methodological principle, we would be pursuing an unreasonable quest for explaining every principle (e.g., see van Fraassen 1981). But as we see in Section 2, it is not clear at all that the geometrical interpretation of special and general relativity is simpler than the dynamical one, if not the opposite. The common symmetries of different matter theories is not analogous with the case of footprints on a sandbeach. In the case of footprints, the theory that posits a foot is likely to be much simpler than a theory that does not (what causes the identical footprints if not a foot?) Note that dynamicists can also explain the common symmetries by fact that all matter fields are coupled with the gravitational field, and again the resulting theory has no fewer theoretic virtues than the geometrical interpretation. Thus I think dynamicists can respond to Norton's first objection by simply rejecting the first horn of Norton's argument.

Norton's second objection is that dynamicism presumes spacetime through its use of coordinate systems. Indeed, the use of coordinate systems is pervasive in dynamicism. For instance, physical quantities and laws are Lorentz invariant just in case their expressions have the same form in a set of coordinate systems related by Lorentz transformation. Norton argues that the use of such coordinate systems is problematic because it presumes spacetime coincidence. For example, consider two clocks of different material constituents and governed by different matter theories recording the same period of time while moving away from each other. We marked one as traveling from parameter $(0, 0, 0, 0)$ to $(x, 0, 0, t)$ in one matter theory, and another as traveling from $(0, 0, 0, 0)$ to $(-x, 0, 0, t)$ in the other matter theory. But to assume that the coordinate parameter $(0, 0, 0, 0)$ in both matter theories refers to the same thing is to introduce spatiotemporal coincidence, which tacitly assumes an underlying spacetime.

Norton preempts two immediate responses. First, he argues that spatiotemporal coincidence cannot be inferred from the coupling—that is, the interaction—of different matter fields. The spins of two particles can be coupled without indicating that they spatiotemporally coincide—it just indicates the correlation of their spin momenta. Second, he rejects the response that we should formulate all matter theories in one set of parameters, in which case a parameter like $(0, 0, 0, 0)$ automatically refers to one and the same thing. The reason is that we cannot simply assume that all matter theories can share a set of parameters without assuming spacetime—after all, we do not assume that all particles share the same internal spin space.

Can we justify the presumption of spatiotemporal coincidence through the universal coupling of matter fields with the gravitational field? Not quite, because in specifying a particular coupling, i.e., how the values of a field correlate to the values of another field at certain points, we already assume that the gravitational field and other matter fields can spread over the same coordinate system (or the same manifold), which is exactly what Norton challenges.

It is worth clarifying again that not all dynamicists reject a substantival spacetime.[15] According to substantival dynamicism, there is a metrically amorphous mani-

[15]Brown's version of dynamicism is ambiguous. Norton reports Brown's feedback to his argument:

> Harvey Brown has assured me that Brown and Pooley ([2004]) and Brown ([2005], especially Ch. 2) presumed the existence of a manifold of spacetime events with coordinate systems... these events and coordinates are posited by Brown and Pooley's texts as spatiotemporal structures, independent of matter, as opposed to ones that can arise only in the presence of matter. Specifically, the analysis shows that they have no

fold that is our spacetime. It follows that GEOMETRICISM-1_{STR} (or GEOMETRICISM-1_{GTR}) is true. So substantival dynamicists only reject the other two principles of geometricism. But to follow this strategy, we have to give up the important benefits of relational dynamicism. To remind: for one thing, if there is a real manifold underlying matter fields, there would be the question whether a spacetime point P is occupied by (say) my laptop or not, which is empirically inaccessible. In that case, the dynamicists would lose their distinct resource to solve the hole argument comparing to the geometricists.

To take stock, Norton objects that if the dynamicists want to do away with manifolds, the use of coordinate systems still tacitly assumes a substantival spacetime. In other words, not all primitive geometrical notions (broadly construed) can be explained away by dynamicists—the notion of spacetime points required for spatiotemporal coincidence is still presumed. This, I think, is an important worry against dynamicism, and dynamicists should appeal to a different framework that neither posits a manifold nor depends on a coordinate space.

4 The Algebraic Approach to Dynamicism

In the standard framework, the coupling (or interaction) between two fields is expressed through defining them on a common spacetime, manifold or coordinate system. This leads to the objection that whatever serves the role of this common background *is* spacetime, and therefore dynamicists cannot consistently hold that spacetime is not part of fundamental reality. To respond to this challenge, it is the best to appeal to a framework that does not construe fields as living on a manifold or

> choice but to make the posit; attempts to extend their constructivism to the manifold of spacetime events will fail. (11-2)

Here, Brown is claimed to endorse the reality of spacetime manifold existing prior to matter. But in the same book, Brown also explicitly denounces spacetime manifold in reply to the hole argument (156).

a coordinate system so that it is clear that spacetime coincidence, along with other chronogeometrical structures, can be derived from rather than being presumed by dynamical laws. A good candidate for such a framework is *algebraicism*, in which fields are elements of algebraic structures without an underlying manifold. Indeed, Menon (2019) proposed a response to Norton based on this approach. However, I will argue that his solution, which is based on traditional algebraic formalism, is seriously flawed. Thus, we must go beyond this formalism. I introduce the new implementation of algebraicism advanced by Chen and Fritz (2021) and discuss how it can help solve the problems.

It is worth highlighting that the formalism of algebraicism wasn't proposed for the purpose of dynamicism. Geroch proposed Einstein algebras in the hope of accommodating quantum theory, which calls for a "smearing-out" of spacetime points. Later, Earman (1989) suggested using algebraicism as a response to the hole argument, though criticized by Rynasiewicz (1992).[16] Algebraicism also provides potentially more flexible framework for physics. For example, we could have algebraic structures that do not correspond to any geometrical structures with points, such as the noncommutative algebras on which noncommutative geometry is built (see Doplicher, Fredenhagen, and Roberts, 1995; Huggett, Lizzi and Menon, 2021). This may be helpful for formulating quantum theories (e.g., see Bain 2003, Heller and Sasin 1999). There is also an added benefit of having a natural metaphysical account of tangent space in this framework (Chen 2022).

4.1 The Traditional Algebraicism and Its Problems

More elaborately, general relativity is standardly formulated in the framework of differential geometry, which features manifolds with topological and differential struc-

[16]Rynasiewicz (1992) points out that algebraicism does not "smear out" spacetime points because points can be reconstructed from the algebraic structure, and it does not avoid the hole argument because the indeterminism among isomorphic algebras is analogous to that among diffeomorphic manifolds. Bain (2003) disputes this claim.

tures. Algebraicism provides an alternative framework for general relativity that does not posit manifolds. Consider a smooth manifold $\mathcal{M}$. Now, consider all the real-valued smooth functions on $\mathcal{M}$. We can operate on these functions in various ways. For example, we can add or multiply two smooth functions to get another smooth function. Now, instead of considering these smooth functions as maps of $\mathcal{M} \to \mathbb{R}$, we can instead consider them abstractly as elements of an algebraic structure $C^\infty(\mathcal{M})$ defined by algebraic operations such as addition and multiplication (for more details, see Geroch 1972, Connes 2013). This algebraic structure is the basic entity postulated by algebraicism. The idea is that all the relevant information about manifolds can be encoded in such algebras of smooth functions. In particular, all geometrical entities that we need to do physics up to general relativity, such as vectors and tensors, can be defined in algebraic terms without any reference to manifolds. In this formalism, the manifold-theoretic structure $\langle \mathcal{M}, g \rangle$ is replaced by $\langle C^\infty(\mathcal{M}), \hat{g} \rangle$ (called "Einstein algebra") where $\hat{g}$ is an algebraic equivalent of the metric field, as solutions to the Einstein field equations (Rosenstock et al. 2015).

We can interpret $C^\infty(M)$ realistically as consisting of all possible configurations of a scalar field (see Earman 1989, Bain 2003). Again, a scalar field configuration here is not a map from manifold points to real numbers, but a simple element in the basic algebraic structure. Under this formalism, we can derive the notion of spacetime coincidence without tacitly assuming spacetime at the fundamental level. For example, consider elements Φ, Ψ of $C^\infty(M)$ which can be respectively realized as a function with value n at point p (and negligible elsewhere) and a function with value m at p (and negligible elsewhere). This spacetime coincidence of Φ, Ψ at p is already encoded in the algebraic structure. For example, let Γ be the element that can be realized as having value $m + n$ at p (and negligible elsewhere), and Ξ be the one realizable as having n at p and m at q (and negligible elsewhere). The spacetime coincidence of the peaks of Φ, Ψ is encoded in the fact that the addition of Φ and

Ψ is Γ rather than Ξ. Menon (2019) proposed a response to Norton based on this approach.

However, this formalism faces serious problems as a solution to Norton's argument against (relational) dynamicism. Recall that two important benefits of dynamicism are that it is more ontologically and ideologically parsimonious, and better avoids empirical underdetermination than the alternatives. However, the current algebraic formalism does not have these virtues—thus this solution is a mismarriage between algebraicism and dynamicism. Let me explain.

According to this approach, there is no spacetime manifold, but there is an algebraic structure $C^\infty(M)$ at the fundamental level. As just mentioned, we can read it realistically as consisting of configurations of a real-valued scalar field. But what is this scalar field? There are no physical scalar fields at the fundamental level acknowledged by current physics—the closest is the Higgs field (a complex-squared-valued scalar field). But even if there is actually a fundamental scalar field, there are two problems. First, this formalism would be privileging one physical field ontologically. For example, following the prescription that all relevant structures are to be defined in terms of the algebra $C^\infty(M)$, non-scalar fields such as the electromagnetic field then would have to be defined in terms of the scalar field, but it is implausible to think that the electromagnetic field is ontologically dependent on the scalar field. Secondly, even if there is an actual scalar field, it's possible that it does not exist, but according to the current formalism, such a field must exist. Moreover, if $C^\infty(M)$ is interpreted as a physical field not acknowledged by current physics, then we would be positing a fundamental "ghost" field that play no role in our best physical theory. Now, this would especially trouble relational dynamicists rather than geometricists or substantivalists. From the latter's perspective, this field is simply the algebraic counterpart of spacetime manifold. But this is not a good option for the dynamicists. Just like spacetime, $C^\infty(M)$ would be a different kind of entity from all other physical fields—no ontological parsimony for dynamicists. The amount of fundamental structure is also equivalent to the manifold approach (Rosenstock et al 2015). Call this approach *substantival algebraicism*.[17]

Alternatively, we can refrain from interpreting $C^\infty(M)$ realistically, as Menon (2019) suggests. $C^\infty(M)$ and its members are considered mathematical abstracta rather than part of physical reality. But this strategy is unsatisfactory too. One reason is that $C^\infty(M)$, treated abstractly, plays an analogous role in this algebraic framework as the coordinate systems that Norton criticizes, and therefore Norton's objection would reemerge for $C^\infty(M)$. Here, all physical fields are defined in terms of $C^\infty(M)$ just as they are defined on a coordinate system in the coordinate-system approach. Like points of a coordinate system, elements of $C^\infty(M)$ are invoked in describing the spacetime coincidence of fields. For example, consider two vector fields which have non-zero values at some points and zero elsewhere. How do we describe whether the non-zero values located at the same region or not? In the coordinate-system approach, we simply compare their values at each point in a coordinate system. Analogously, in the algebraic framework, we compare how they act on $C^\infty(M)$.[18] Thus, we should avoid the explicit reference to $C^\infty(M)$ just as we should avoid resorting to coordinate systems.

Next, if we interpret $C^\infty(M)$ realistically, the problem of empirical underdetermination would also reemerge in this formalism. In the manifold-theoretic case, the hole argument is premised on that two distinct models related by diffeomorphisms, such as $\langle \mathcal{M}, g\rangle$ and $\langle \mathcal{M}', g'\rangle$ where g is carried to g' by a diffeomorphism from $\mathcal{M}$ to $\mathcal{M}'$, can

[17] Some people mean by "substantival algebraicism" that every element of the algebra is interpreted realistically, which I endorse in this paper. But here it means that the interpretation of the fundamental entities in the algebra is like spacetime. See Bain 2003, Earman 1989.

[18] A vector field is a collection of *derivations* which are maps from $C^\infty(M)$ to $C^\infty(M)$ that satisfy various conditions (see Geroch 1972, 272). Here's how we compare the values of two vector fields mentioned in the text. The non-zero values of the two fields Ψ, Φ locate at the same region just in case for all $f \in C^\infty(M)$, $f\Psi = 0 \leftrightarrow f\Phi = 0$, where $f\Psi$ is defined by $(f\Psi)(g) = f\Psi(g)$ for all $g \in C^\infty(M)$. The idea is that, if the regions that the two fields have non-zero values are the same, then we can "detect" this by multiplying both fields by all functions that are zero at that region and see if they behave the same.

solve the Einstein field equations equally well. Thus, there are distinct models that are empirically and dynamically indistinguishable, which is a problem. But now we can construct just as well two distinct models $\langle C^\infty(\mathcal{M}), \hat{g}\rangle$ and $\langle C^\infty(\mathcal{M}'), \hat{g}'\rangle$ that satisfy the algebraic Einstein equations. For concreteness, we can consider $\mathcal{M}'$ as a proper subset of $\mathcal{M}$ in both cases, and g' is the restriction of g to $\mathcal{M}'$. Then $C^\infty(\mathcal{M}'$ consists of elements of $C^\infty(\mathcal{M})$ restricted to $\mathcal{M}'$ and $\hat{g}'$ is defined on $C^\infty(\mathcal{M}')$ in the same way as how $\hat{g}$ is defined on $C^\infty(\mathcal{M})$. It follows that we have distinct algebraic models (related by homomorphism) that are empirically and dynamically indistinguishable. This is just one more symptom of the problem that the algebraic structure $C^\infty(\mathcal{M})$ plays the same role as the spacetime manifold. Physical laws only specify how fields are defined on a manifold only up to diffeomorphism just as they specify how fields are defined on the basic algebraic structure up to homomorphism. Therefore, this formalism cannot help dynamicists answer the hole argument.

There is another problem when we formulate the algebraic approach in the set-theoretic framework, recognized already by Earman (1989). Insofar as we have a principled problem of choosing between isomorphic manifold-theoretic models, we have an analogous problem of choosing between isomorphic algebraic models. For example, we might think the algebraic model built on the set of smooth functions on $\mathbb{R}$ is distinct from the model built on the set of smooth functions on $(0, 1)$ simply because the two sets have different elements, even though it seems only reasonable to say that these are just two mathematical ways of talking about the same physical situation. We always have this problem in the set-theoretic framework regardless of how the structures are built upon the sets or how physical laws are formulated.

4.2 dynamical algebraicism

To address these problems, we need to go beyond the traditional formalism of algebraicism. I will propose a solution called *dynamical algebraicism* based on the formalism proposed by Chen and Fritz (2021). Their approach appears to have the following distinct features: (1) it does not privilege a particular type of physical fields or posit a "ghost" field that plays the role of spacetime, providing structure for other physical fields. The ontology consists of only fundamental fields recognized by physics. (2) The structure of the physical fields postulated is restricted to what is required to do physics. This is to avoid the situation that, for example, two physical fields can be related in distinct ways that make no difference to physics. (3) The formalism is formulated in category theory, which helps avoid the general problem of having distinct isomorphic models formulated in standard set theory. According to the authors, this framework is still very preliminary for doing physics, but because of these features, it seems a step in the right direction for the dynamical approach to physics. The upcoming discussion will presuppose some basic familiarity with category theory and therefore not include definitions of basic technical terms.[19]

To meet the first challenge, instead of positing $C^\infty(M)$ as the basic structure, we can let the basic algebraic structure consist of all physical fields recognized by current physics. But as we know, there are many types of physical fields with diverse mathematical representations: e.g., the electromagnetic field is a one-form, fermions fields are spinor fields, and the metric field is a tensor field. So how can we put them into one algebraic structure? Here's the trick facilitated by category theory. First, we conceptualize a physical field as a *field functor* $\mathcal{F}$ (equipped with a Lagrangian encoding its dynamical information). In pre-algebraic terms, a field functor is a functor from a customizable category of "spacetime" (in which the objects are certain representations of spacetime such as manifolds and the maps are certain transformations between them such as diffeomorphisms) to category of sets, assigning to every (say) manifold a set of field configurations, which commutes with diffeomorphisms between manifolds. The reason why we would want the assignments to commute

[19]If the readers are not familiar with basic category theory, I shall recommend Awodey's (2010) introductory book to category theory for an explanation of relevant concepts.

with diffeomorphisms (that is, why fields should be conceptualized as functors) is that a physical field should be invariant under changes of coordinates (in the general sense that any transformations that do not make differences to physics are considered mere coordinate changes). To see the connection, let's consider a simple example: consider the category that consists of all coordinate representations of Minkowski spacetime as its objects and all Lorentz transformations as its maps. A real-valued scalar field, then, is a functor that specifies a smooth function for every coordinate representation of Minkowski spacetime that commutes with Lorentz transformation. This way, we build the Lorentz invariance into the description of the field (the scalar field is not represented by a single smooth function but all of them related by Lorentz transformations).

To quickly forestall a common confusion, the appearance of "spacetime" in the formulation of field functors is only provisional, and will be eliminated when the functors are reconceptualized as elements of the basic algebraic structure, as I will explain soon. (In the same sense, the appearance of $\mathcal{M}$ in $C^\infty(\mathcal{M})$ under the traditional algebraicism is also provisional.)

Suppose the physical field of interest include fields of different formal types like scalar fields and tensor fields (such as the metric field).[20] How do we reconceptualize their field configurations as simple elements in a unified algebraic structure that does not include any other elements? In category theory, a *natural transformation* is a "higher-order functor" from the functors from one category to another that preserves the functors' behavior. Just as the smooth functions on manifolds can encode all information of manifolds in the traditional algebraicism, natural transformations between functors can potentially encode all information about the functors. Furthermore, we can define *natural operators* in terms of natural transformations (the only difference between them is that a natural transformation is a binary operator while

[20] The inclusion of scalar fields just serves as a simple example. The field algebra does not have to include a scalar field. See the example of spinor electromagnetism in Chen and Fritz (2021).

a natural operator can be n-ary). Then we can reconceptualize the field functors as elements of the *field algebra* characterized by all the natural operators on the field functors. (For details on natural operators, see Kolar et al. 1993; for examples of field algebras, see Chen and Fritz 2021)

It is worth emphasizing that what natural operators we postulate depends on the details of what functors we use to represent various fields. For a given field, there is a great deal of flexibility in customizing its functor in order to achieve the desired invariance of fields—that is, to minimize distinctions without physical difference. For example, an electromagnetic field (represented by a 1-form tensor field) is invariant under gauge transformation: adding the gradient of a scalar field to the electromagnetic field does not correspond to any physical change (see Healey 2007 for a philosophical exposition of gauge invariance).[21] Accordingly, we can formulate functors in a way that identifies field configurations that are related by gauge transformations: instead of assigning all possible configurations of 1-form to a given manifold, we should assign only *the quotient set* of those configurations that does not distinguish between ones differing only by the gradient of a scalar field. This affects what natural operators we can define on the field.

Natural operators are abundant and some may not correspond to physical reality. To avoid distinctions without difference, we should pare down the natural operators to those needed for physics so that we do not have redundant structures in the field algebra that do not have physical significance.[22] For example, in order to formulate the Lagrangian of a scalar field ϕ according to a particular theory (such as ϕ^4-theory), we need an operator on the scalar field that gives the "length" of the gradient of ϕ, and this operator is one of the natural operators that characterize the field algebra, in

[21]The notion of gradient is similar to that of (total) derivative, but is more commonly used in the context of physical fields in space with more than two dimensions.

[22]I do not claim—neither do the authors—that this framework will avoid redundant structures or distinctions without difference completely, which they do not know. But trying not to postulate physically superfluous algebraic structures is a step in the right direction.

which ϕ is an element.[23] But there are a vast number of natural operators on the field that are not required by physics.[24] If we do not pare down natural operators to those that are required by physics, a homomorphism from one field algebra to another that preserves all physically-required operators but not all natural operators can generate distinct models that presumably does not correspond to genuine physical difference. Therefore, it is important that we construct field algebras just with enough natural operators that are required for physics.

Finally, this approach is formulated within category theory, which does not distinguish between isomorphic structures. For example, recall that when we give set-theoretic constructions of a manifold, we can define a topological and differential structure on a set of spacetime points, or a set of singleton of points, or a set of real numbers—the resulting constructions would be distinct because set members are distinct. In contrast, the "internal" structures of sets that do not respect diffeomorphisms between manifolds are not expressible in the category of manifolds. Thus we cannot distinguish between diffeomorphic manifolds. Similarly, we cannot construct two distinct isomorphic field algebras within the same category of interest. (For more discussions on category theory and structuralism, see for example Awodey 1996, Bain 2013)

Let's take stock. Since it has been shown by Chen and Fritz that this formalism is feasible for doing physics at least to a preliminary degree, and since that it posits no substantival spacetime or its substitutes, we can use it as a preliminary response to Norton's objection that dynamicism must presume substantival spacetime

[23]According to the massless scalar ϕ^4-theory in d dimensions—a commonly used toy example in physics—the Lagrangian density is given by $\mathcal{L}^{\text{scalar}}(\phi) = \left(\frac{1}{2}g^{\mu\nu}\left(\partial_\mu\phi\right)\left(\partial_\nu\phi\right) - \frac{\lambda}{4!}\phi^4\right)\sqrt{|\det g|}$. Here, the commutative binary operation $\langle d-, d-\rangle$ on $\phi \times \phi$ is a natural operation that gives us the "length" of the gradient of ϕ.

[24]For example, every smooth operator equipped by C^∞-*rings* is a natural operator on a scalar field. (For every n-argument smooth function on $\mathbb{R}$, there is a corresponding smooth operator on a C^∞-ring that takes its n elements to another element. For more information on C^∞-rings, see for example Moerdijk and Reyes 1991.) But what we need for physics up to general relativity is arguably only *commutative rings*, equipped with addition and multiplication (Geroch 1972).

to account for spacetime coincidence. Unlike the traditional algebraic solution, the elements in the field algebra are actual physical fields with energy and momentum and interactions. There is no tacit arena that physical fields live on, and the dynamical interactions of physical fields defined by natural operators are treated as primitive, from which spacetime coincidence and other geometrical notions may arise.

Like standard dynamicism, there is no primitive chronogeometrical significance of the gravitational field beyond the field equations. If we restrict to special relativity, there is similarly no need for a separate gravitational field because the metric information can be encoded in the algebraic structure of the field algebra (Chen and Fritz 2021, 25-26). The hole argument is unlikely to rise because special care has been given to that the basic algebraic structures are defined through physically relevant operators and that we do not distinguish between isomorphic structures. Therefore, I submit that dynamical algebraicism is an attractive implementation of dynamicism and an adequate response to Norton's worries.

5 Conclusion

In this paper, I have motivated the dynamical approach to relativistic theories by highlighting how it addresses the shortcomings of the geometrical approach. I also defend it against Norton's objection by appealing to a new algebraic implementation of dynamicism called *dynamical algebraicism*, which I argue to be an improvement over the solution based on the traditional implementation of algebraicism.

According to this approach, there are no manifolds or any fundamental entity that plays the role of a substantival spacetime. Physical fields and only physical fields are fundamental, which are not defined on manifolds or coordinate systems but through the natural operators among them. Special attention is paid to positing only structures with physical significance, so we are in a better position to answer challenges from empirical underdetermination and radical indeterminism. In particular, since there is no substantival spacetime or its substitute, there is no objective reality about how the metric field and matter fields spread over it, and the hole argument does not arise.

As a dynamical approach, the metric field in general relativity is treated as one of the matter fields that obeys dynamical laws. There is no spacetime geometry prior to the dynamical laws. In particular, the chronogeometric-dynamical laws are not posited as basic, but as approximate principles derivable from the field equations. This results in a more ontologically and ideologically parsimonious theory with more flexibility and fewer unwarranted restraints on the behavior of matter.

References

[1] Acuña, P. (2016). "Minkowski spacetime and Lorentz invariance: The cart and the horse or two sides of a single coin?" *Studies in History and Philosophy of Modern Physics* 55:1-12.

[2] Awodey, S. (1996) "Structure in mathematics and logic: A categorical perspective," *Philosophia Mathematica* 4 (3):209-237.

[3] Awodey, S. (2010) *Category Theory* Oxford, England: Oxford University Press.

[4] Bain, Jonathan (2003). "Einstein algebras and the hole argument". In: PSA 2002. Part I. Vol. 70. University of Chicago Press, pp. 1073–1085

[5] Bain, J. (2013). "Category-theoretic structure and radical ontic structural realism." *Synthese*, 190, 1621–1635.

[6] Brown, Harvey (2005). *Physical Relativity: Space-time structure from a dynamical perspective*. Oxford University Press, New York, NY.

[7] Brown, H., Pooley, O. (2004) "Minkowski space-time: a glorious non-entity." *Philosophy and Foundations of Physics* 1, 67-89. arXiv:physics/0403088.

[8] Brown, H., Read, J. (2021) "The Dynamical Approach to Spacetime Theories," In Eleanor Knox & Alistair Wilson (eds.), *The Routledge Companion to Philosophy of Physics*.

[9] Burgess, C.P. (2003). "Quantum Gravity in Everyday Life: General Relativity as an Effective Field Theory," *Living Reviews of Relativity*.

[10] Chen, L. (2022) "Smooth Infinitesimals in the Metaphysical Foundation of Spacetime theories." *Journal of Philosophical Logic*.

[11] Chen and Fritz (2021) "An algebraic approach to physical fields," *Studies in History and Philosophy of Science* 89, 188-201.

[12] Connes, Alain (2013). "On the spectral characterization of manifolds". In: *J. Noncommut. Geom.* 7.1. arXiv:0810.2088, pp. 1–82.

[13] Crowther, K. (2013) "Emergent spacetime according to effective field theory: From Top-down and Bottom-up" *Studies in History and Philosophy of Modern Physics Part B* 44(3). 321-328.

[14] Doplicher, Sergio, Klaus Fredenhagen, and John E. Roberts (1995). "The quantum structure of spacetime at the Planck scale and quantum fields". In: *Comm. Math. Phys.* 172.1. arXiv:hep-th/0303037, pp. 187–220.

[15] Dray, Ellis, Hellaby, Manogue (1997) "Gravity and Signature Change" *General Relativity and Gravitation* 29: 591-597.

[16] Earman, John (1989). *World enough and space-time: Absolute versus relational theories of space and time*. MIT Press, Cambridge, MA, pp. xvi+233.

[17] Earman, John and Norton, John (1987). "What price spacetime substantivalism? The hole story". In: *British J. Philos. Sci.* 38.4, pp. 515–525.

[18] Eppley, K., Hannah, E. (1977) "The necessity of quantizing the gravitational field," *Foundations of Physics* 7 (1-2):51-68.

[19] French and Ladyman (2003) "Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure," *Synthese* 136(1): 31-56.

[20] Friedman, M. (1983) *Foundations of Space-time Theories: Relativistic Physics and Philosophy of Science*. Princeton University Press.

[21] Geroch, Robert (1972). "Einstein algebras". In: *Comm. Math. Phys.* 26, pp. 271–275.

[22] Healey, R. (2007) *Gauging What's Real: The Conceptual Foundations of Contemporary Gauge Theories.* Oxford University Press.

[23] Heller, Michael and Wieslaw Sasin (1999). "Noncommutative unification of general relativity and quantum mechanics". In: *Internat. J. Theoret. Phys.* 38.6, pp. 1619–1642.

[24] Hoefer (1996) "The Metaphysics of Space-Time Substantivalism." *The Journal of Philosophy*, Vol. 93, No. 1, pp. 5-27.

[25] Huggett, N., Lizzi, F., & Menon, T. (2021). "Missing the point in noncommutative geometry." *Synthese* 199, 4695–4728.

[26] Janssen, M. (2002) "COI Stories: Explanation and Evidence in the History of Science." *Perspectives on Science* 10 (4):457-522 (2002)

[27] Knox (2017) "Physical Relativity from a Functionalist Perspective," *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics* 67.

[28] Krasnov, Kirill (2020). *Formulations of General Relativity: Gravity, spinors and differential forms.* See also pirsa.org/19020079/. Cambridge University Press.

[29] Kolar, Ivan, Peter W. Michor, and Jan Slovak (1993). *Natural operations in differential geometry.* Springer-Verlag, Berlin, pp. vi+434.

[30] Malament, D.B. (2012). "A remark about the 'geodesic principle' in general relativity." In *Analysis and Interpretation in the Exact Sciences*, pages 245–252.

[31] Martínez (2007) "There's No Pain in the FitzGerald Contraction, Is There?" *Studies in History and Philosophy of Modern Physics*, Vol. 38, pp. 209-215.

[32] Mattingly, J. "Singularities and scalar fields: Matter theory and general relativity," *Proceedings of the Philosophy of Science Association* 2001 (3):S395-.

[33] Maudlin, T. (1993). "Buckets of water and waves of space: Why spacetime is probably a substance." *Philosophy of Science* 60 (2), 183–203.

[34] Maudlin, T. (2012). *Philosophy of Physics: Space and Time*. Princeton University Press.

[35] McKenzie, K. (2014) "Priority and Particle Physics: Ontic Structural Realism as a Fundamentality Thesis," *British Journal for the Philosophy of Science* 65 (2):353-380.

[36] Menon, T. (2019). "Algebraic fields and the dynamical approach to physical geometry". In: Philos. Sci. 86.5, pp. 1273–1283.

[37] Menon, T., Linnemann, N., & Read, J. (2020). "Clocks and chronogeometry: rotating spacetimes and the relativistic null hypothesis." *British Journal for the Philosophy of Science*, 71(4) 1287-1317.

[38] Misner, Thorne and Wheeler (1973). *Gravitation*. Princeton University Press.

[39] Moerdijk, I., Reyes, G. (1991) *Models for Smooth Infinitesimal Analysis*. Springer-Verlag New York.

[40] Norton, J.D. (2008) "Why Constructive Relativity Fails?" *British Journal for the Philosophy of Science.* 59 (4): 821-834.

[41] Pooley, O. (2006) "Points, particles, and structural realism," *The structural foundations of quantum gravity*, 83-120.

[42] Rosenstock, S., Barrett, T., Weatherall, J. (2015) "On Einstein Algebras and Relativistic Spacetimes," *Studies in History and Philosophy of Modern Physics* 52: 309-315.

[43] Rovelli, C. (1997) “Halfway through the woods: contemporary research on space and time,” in John Earman and John D. Norton, (eds.), *The Cosmos of Science*, 180–223. University of Pittsburgh Press, Pittsburgh.

[44] Rovelli, C. and Vidotto, F. (2015) *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity, and Spinfoam Theory*. Cambridge University Press, Cambridge.

[45] Rynasiewicz, Robert (1992). “Rings, holes and substantivalism: on the program of Leibniz algebras”. In: *Philos. Sci.* 59.4, pp. 572–58.

[46] Tamir, M. (2012) “Proving the principle: Taking geodesic dynamics too seriously in Einstein's theory,” *Studies in History and Philosophy of Modern Physics* 43 (2):137-154 (2012)

[47] Van Fraassen, C. B. (1980) *The Scientific Image*. Oxford University Press.

[48] Weatherall, J.O. (2011). “On the status of the geodesic principle in newtonian and relativistic physics.” *Studies In History and Philosophy of Modern Physics*, 42(4):276–281.