Abstract. Dynamicism is the view that the dynamic laws are explanatorily more fundamental than the geometric feature of the world. In the context of general relativity, the chronogeometric significance of the metric field—considered as a matter field—is explained by its dynamic laws (Brown 2005). The opposite view that geometry is fundamental and presumed by dynamics is called geometricism (e.g., Friedman 1995, Maudlin 2012). In this paper, I renew the support for dynamicism with the reason that positing (chrono)geometric principles as fundamental not only leads to more complicated ontology and ideology but also imposes unwarranted restrictions on the behaviors of matter. I also revisit and respond to two objections to dynamicism by Norton (2008). While I adopt algebraicism (a framework that does not posit a manifold) as a response, I argue that Menon’s (2019) response based on a traditional formalism of algebraicism is seriously flawed. Instead, I suggest a solution—which I call dynamic algebraicism—based on the new formalism by Chen and Fritz (2021).

Keywords. Physical relativity; dynamicism; geometricism; algebraicism; Einstein algebras; field algebras; natural operators.

1 Introduction: Geometricism vs Dynamicism

Does space (or spacetime) exist? This question is the central focus of the traditional debate between absolutism and relationalism. Absolutism says that space exists as
the container of matter and material processes, while relationalism says that it is reducible to relations between material bodies. The modern development of relativistic theories seems to adjudicate in favor of absolutism (for example, see Friedman 1983). According to the standard interpretation of general relativity, spacetime not only exists but has a curvature, which can be formulated as a metric field and is determined by the distribution of mass-energy. Even if relativists disagree, they still need to include the metric field in their ontology that plays the role of spacetime—though interpreted as a matter field—and their resulting position does not seem interestingly different (Maudlin 1993).

In light of this, the old debate is partly transformed into a new one between geometricism and dynamicism. The new debate includes the ontological claims about spacetime but emphasizes more on the ideological status of geometric (or chronogeometric) notions and laws. According to geometricism, the geometric features of our world are fundamental and exist independently from (and are presumed by) dynamic laws. According to dynamicism, it is opposite: the dynamic laws are fundamental and the geometric features and laws are derived from them. In the context of general relativity, geometricists hold that the metric field represents the fundamental geometric feature of spacetime, while dynamicists hold that the (chrono)geometric significance of the metric field (as a matter field) is derived from the dynamics of measuring devices like rods and clocks that survey the field. Note that in the traditional debate between absolutism and relativism, both sides typically assume geometricism: for example, both Newton and Leibniz considered the notion of spatial distances to be fundamental.

Geometricism has been the more standard position in the literature, advocated by authors like Maudlin (2012), Earman and Norton (1987), and Friedman (1995). Like absolutists, the geometricists appeal to the explanatory role played by the primitive geometric feature of spacetime. For example, clocks behave similarly despite their
different matter constitutions because they track the same spacetime intervals. On
the other hand, Brown (2005) has famously argued for dynamicism, denying the ex-
planatory power of the spacetime geometry. In this paper, I shall distinguish between
two versions of dynamicism as follows:

**DYNAMICISM WITH MANIFOLD (Weak).** A metrically amorphous space-
time manifold exists fundamentally. But its geometric features are derived
from dynamic laws governing matter fields.

**DYNAMICISM WITHOUT MANIFOLD (Strong).** Physical fields governed
by dynamic laws exist fundamentally without an underlying manifold and
its geometric structure.

In this paper, I will argue for dynamicism against geometricism, and also argue in
favor of **Strong** over **Weak**.¹ That is, I will argue that there is no spacetime
geometry at the fundamental level and the geometric significance of the metric field
is derived—not only so, but the metrically amorphous manifold commonly assumed
also does not exist.

The debate sometimes reaches a stalemate on what explains what and what needs
to be explained. That is, geometricists (for example, Maudlin 2012) insist that it is
spacetime geometry that explains the common features of dynamic laws, while dy-
namicists insist that it is the chronogeometric-dynamic laws (namely laws that specify

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¹It is unclear which version Brown (2005) held, since the following quotes point to different
answers:

The simplest (and to my mind the best) conclusion, and one which tallies with our
usual intuitions concerning the gauge freedom in electrodynamics, is that the space-
time manifold is a non-entity. (Brown 2005, 156)

Harvey Brown has assured me that Brown and Pooley ([2004]) and Brown ([2005],
especially Ch. 2) presumed the existence of a manifold of spacetime events with co-
ordinate systems... these events and coordinates are posited by Brown and Pooley’s
texts as spatiotemporal structures, independent of matter, as opposed to ones that can
arise only in the presence of matter. (Norton 2008, 11-2)

It may be inferred from these texts that Brown (2005) did not settle on whether manifold exists. I
will return to this point in Section 3.
how physical systems behave in accordance with spacetime geometry) assumed by geometricists that need to be explained. A mere insistence is certain not enough (see Knox 2017, Norton 2008). I aim to renew the support for dynamicism by arguing that holding (chrono)geometric laws to be fundamental not only leads to a more complex ontology and ideology but also imposes unwarranted restrictions on the behaviors of matter (Section 2).

After explaining the advantages of dynamicism, I will defend the view from Norton’s (2008) objections (Section 3). His two objections target at the two versions of dynamicism respectively (though Norton himself did not distinguish between them). As I shall argue, Norton’s objection against Weak is weak but the one against Strong is strong, which is why I think that dynamicists should go beyond the standard framework of differential geometry. Roughly, Norton argues that dynamicists must presume at least the notion of spacetime coincidence, which is a geometric notion, and thus are mistaken in holding that geometry is entirely derived from dynamics. To adequately address this objection, we need a technically feasible framework that deals away with manifolds or an underlying arena that physical fields live on.

A good candidate for such a framework comes from algebraicism. A core feature of an algebraic framework is that it does not posit manifolds but replace them with algebraic structures. For example, in the standard formalism proposed by Geroch (1972), all information about a manifold is encoded in the algebraic structure consisting of all smooth functions on the manifold. These functions are no longer analyzed as maps from the manifold to real numbers, but are reconceptualized as simple elements in the algebraic structure characterized by all the algebraic operations we can perform on them. Geroch shows that we can do physics with them up to general relativity. Menon (2019) proposes a response to Norton based on this formalism, pointing out that this can help make sense of spacetime coincidence without assuming spacetime points (since there is no manifold). However, I think this particular marriage be-
tween dynamicism and this algebraic formalism is a mistake, because this algebraic formalism has serious flaws as a dynamic solution (Section 4.1). For one thing, the most natural physical interpretation of the basic algebraic structure posited there is spacetime. Indeed the position is sometimes called “algebraic substantivalism” (see Earman 1989, Bain 2003; also mathematicians sometimes call the algebraic structure “spacetime”). It does not correspond to any matter field recognized by fundamental physics and carries no energy and momentum. It is just another representation of (metrically amorphous) spacetime traditionally represented by a manifold.

Instead, I introduce a new algebraic response to Norton’s argument based on Chen and Fritz’s (2021) formalism of field algebras, which I call dynamic algebraicism (Section 4.2). According to this approach, all physical fields are treated on a par, and the basic algebraic structure consists of only physical fields that are acknowledged by current physics (which can be expanded if more are discovered). The technical challenge this approach faces is to codify different types of physical fields (for example, a one-form field, a tensor field of rank (0,2), a spinor field) within a single algebraic structure. Recall that in Geroch’s formalism, all elements of the basic algebraic structure are smooth functions. This challenge can be met by using the technique from category theory (a framework known for its power of generality). Very roughly, a field is first conceptualized as a functor from the category of manifolds to the category of sets that assign field configurations to manifolds. Then, fields of different types are reconceptualized as elements of an algebraic structure characterized by natural operators on field functors. I argue that the dynamic solution based on this formalism can meet the challenges that trouble the other approaches.

Let me briefly remark on the relevance of this debate between geometricism and dynamicism for other contemporary issues in philosophy of physics. Even though general relativity is not a final theory of spacetime due to its conflict with quantum theory, the discussion on the interpretation of relativistic theories is not obsolete.
Precisely because there are lots of obscurities around how to develop quantum gravity (a research program unifying general relativity and quantum theories), thinking more clearly about relativity can be important (after all, we can use some help from all angles). Moreover, I think there is a rather intimate relation between quantum theories and dynamicism given that “spacetime” is expected to undergo quantum effects like other matter fields (Section 2.3). The algebraic framework can also lead to more freedom in formulating quantum gravity.

2 The Case for Dynamicism

I will argue that dynamicism, the view that all (chrono)geometric properties and relations are explained by fundamental dynamic laws, is better than geometricism, which says otherwise. To engage with the literature, I will primarily focus on relativistic theory. I will first briefly lay out the basic principles of general and special relativity according to geometricism and dynamicism respectively, to prepare for my arguments in favor of dynamic interpretation of relativity. I also argue towards the end of the section that going beyond relativity further bolsters the case for dynamicism.

2.1 Geometricism

Geometricism says spacetime exists independently of matter, and has an intrinsic geometry that plays a role in determining how matter behaves. The geometric interpretation of general relativity can be formulated into the following principles (in the style of Norton 2008):

It might be worth mentioning that I argue elsewhere that we can use quantum physics to argue for dynamicism in discrete spacetime, which is a particularly conceptually clear case for dynamicism: the apparent geometry emerges from the dynamic laws defined on discrete space devoid of geometric structures ([Anonymized]; also see van der Vaart 1998 and Montvay and Münster 1994).

For instance, the formalism of noncommutative geometry in the algebraic framework cannot be easily interpreted in terms of manifold points (see Huggett et al. 2021).

Norton formulated the geometric interpretation of special relativity into three similar principles, which include GEOMETRICISM-3. The other two are:
Geometricism-1. There exists a spacetime that can be coordinated by a set of standard coordinates \((x, y, z, t)\) that are related to each other by general coordinate transformations.

Geometricism-2. Metric is an intrinsic property of spacetime independent of the process it contains, and is (partially) determined by the distribution of mass-energy in the way specified by the Einstein field equations. The spatiotemporal intervals surveyed by ideal clocks and rods are determined by the metric.

Geometricism-3. Material clocks and rods measure spatiotemporal intervals because there are chronogeometric-dynamic laws that specify the relation between the geometry of spacetime and the behaviors of matters.

The chronogeometric-dynamic laws mentioned in Geometricism-3 are typically expressed as the following three principles:

The Null Hypothesis. The trajectory of light is a null geodesic regardless of the physical state of its source.

The Geodesic Principle. The trajectory of any free system is a timelike geodesic.

The Clock Hypothesis. The amount of time recorded by an ideal clock is (proportional to) the length of its worldline.

Geometricism-1_{STR}. There exists a spacetime that can be coordinated by a set of standard coordinates \((x, y, z, t)\) that are related to each other by Lorentz transformation.

Geometricism-2_{STR}. The spatiotemporal interval \(I\) between any two spacetime points \((x_1, y_1, z_1, t_1)\) and \((x_2, y_2, z_2, t_2)\) is an intrinsic property of spacetime independent from processes it contains, and is given by \(I^2 = (t_1 - t_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 - (z_1 - z_2)^2\).

If \(I^2 > 0\), then \(I\) is time-like, which corresponds to the time elapsed on an ideal clock traveling straight from one point to the other, and if \(I^2 < 0\), then \(I\) is space-like, which corresponds to the distance measured by an ideal rod.
The last principle involves the notion of “clock” which may not seem a fundamental notion. (As Maudlin (2012) puts it: “Nature may recognize a distinction between light and massive particles, but Nature does not have to settle whether a given mechanism counts as a ‘clock’ in order to determine how it should behave.” (106)) To make it more precise, an ideal clock is just a system the state of which changes periodically, and when non-accelerating, each period corresponds to an equal spacetime interval.\textsuperscript{5} Together these chronogeometric-dynamic laws specify how matter behaves in accordance to spacetime geometry.

\subsection*{2.2 Dynamicism}

Dynamicism is the view that spacetime geometry is not fundamental and can be derived from the dynamic laws governing matter. In the case of special relativity, the fundamental principle is simply that the dynamic laws are all \textit{Lorentz invariant}. This means that the dynamic laws have the same form in coordinate systems related to each other by Lorentz transformation. To highlight:

\begin{center}
\textbf{DYNAMICISM}_{STR}. All physical laws are Lorentz invariant.
\end{center}

This principle is sufficient to ground all empirical implications of special relativity. To see it, we can recover the chronogeometric-dynamic laws from this one—not as pure postulates about physical reality but as partially our constructions (indeed, Norton calls dynamicism “constructivism”).

\textsuperscript{5}This may seem like an untestable principle because we cannot test whether an actual system behaves like an ideal clock by directly measuring how long each period of a clock lasts (which we have no access to without circularity). But we can compare the periods of different clocks. If their periodicities are all consistent relative to each other, then they can be considered accurate, because according to geometricists, the best explanation for their synchronization is that they all correctly track spacetime intervals.

Of course, no actual clock is perfectly ideal. In practice, we use atomic clocks as the standard measurement of time. It is the successor to the standard of time called ephemeris time, which is based on Earth’s orbit around the sun. Ephemeris time was abandoned due to random fluctuations of earth that make this standard unreliable. A well-built caesium atomic clock still has the risk of abnormality, so the current standard of time is actually based on the average of many such clocks in different laboratories (see Brown 2005).
First, the null hypothesis can be recovered from the principle that the speed of light (or the Maxwell equations) is Lorentz invariant (this is known to be one of the foundational principles of special relativity), because this principle allows us to *construct* a Minkowski spacetime in which the trajectory of light is a null geodesic. So, Law of Light is essentially reduced to the Lorentz invariance of speed of light, which is just an instance of Dynamicism$_{STR}$ applying to electromagnetism. The geometric notions such as “Minkowski metric” and “null geodesics” are our constructions and do not correspond to fundamental reality.

Similarly, the geodesic principle is reduced to that the physical laws in various matter theories are Lorentz invariant. Again, this permits us to construct a Minkowski spacetime with its timelike geodesics being the trajectories of free-falling material bodies. There are no antecedently existent geodesics that coincide with such trajectories. The clock hypothesis amounts to that the laws governing the matter that constitutes an ideal clock are Lorentz invariant (in addition to the conditions for being a clock$^6$). We can recover the principle by defining the spatiotemporal interval of the worldline of an ideal clock to be the time it records.$^7$

In the case of general relativity, the metric field is indispensable, and dynamicists cannot explain it away through the symmetries of other matter fields as in special relativity.$^8$ Thus, like geometricism, dynamicism needs to include the Einstein field

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$^6$ For example, the matter of a clock needs to have a time-like worldline in order to record proper time (in the geometric terms).

$^7$ A caveat: we need to appeal to clocks that function normally not only when moving inertially but also when accelerating (this is also true for geometricism). That is, when a clock is accelerating, we need to make sure that its periodicity is not affected by its acceleration (such disturbances, if exist, can be discovered in the same way as we discover the abnormality of an atomic clock or ephemeris time). This boils down to the requirement that the restorative force of a clock is stronger than the acceleration force. As Brown (2005) points out, this is analogous to the dynamic understanding of length in Euclidean space. According to dynamicism, the length of an object is defined by the behaviors of measuring devices such as rigid rods or strings. Whether such measuring devices still track length when they are bent depends on whether bending distorts the relevant properties of their constituents such as the comparative lengths of their atomic links.

$^8$ Of course, there are alternative equivalent formulations of general relativity that do not involve a metric field (see Krasnov 2020). But in those formulations, some other manifold-theoretic structures replace the metric field, such as differential forms, connections, or spins. It does not seem that switching to these structures will affect our discussions in important ways—we still face the same
equations as basic postulates of general relativity. But unlike geometricism, the metric field is treated as one of the matter fields and its chronogeometric significance is considered as derived from its interactions with other matter fields.\textsuperscript{9} Since the notion of “metric” has the geometric connotation, perhaps indicating that it is a mathematical device for representing geometric properties of spacetime, I will call the field \textit{the gravitational field} in the context of dynamicism. Similar to GEOMETRICISM-2, we have:

\begin{center}
\textbf{DYNAMICISM.} The gravitational field is coupled with the distribution of mass-energy in accordance to the Einstein field equations.
\end{center}

It is not completely settled in the literature what other basic principles should be postulated in the dynamic interpretation of general relativity. Unlike in the case of special relativity, the chronogeometric-dynamic laws cannot be straightforwardly recovered from \textit{Dynamicism} (although they can \textit{nearly} be recovered!).\textsuperscript{10} However, I will argue in the upcoming discussion that this is an advantage of dynamicism rather than a problem.

\section{2.3 Why dynamicism is better}

I will argue that the dynamic interpretation of relativistic theories is better than the geometric interpretation. In particular, I will argue that unlike geometricism, dynamicism does not impose unnecessary constraints on matter, allows a simpler ontology and ideology and avoids some empirical underdetermination that troubles \textsuperscript{interpretative problems.}

\footnote{Note that if we interpret the metric field as the geometric property of other physical fields rather than of spacetime—which is analogous with assigning spacetime intervals as relations between events or material bodies in special relativity—this would not be a dynamicist view even if we do not posit spacetime. Again, the debate between geometricism and dynamicism is less about the ontology of spacetime than about the explanatory order between geometric and dynamic laws.}

\footnote{It might be worth emphasizing again that we are not aiming to recover the principle as literally true, since there is no spacetime according to dynamicism, but only as to the effect that we can construct a spacetime that has the trajectories of free falling systems as its time-like geodesics. The same applies to other chronogeometric laws.}

\textsuperscript{9}It might be worth emphasizing again that we are not aiming to recover the principle as literally true, since there is no spacetime according to dynamicism, but only as to the effect that we can construct a spacetime that has the trajectories of free falling systems as its time-like geodesics. The same applies to other chronogeometric laws.
geometricism. Finally, I will argue that the case beyond relativity further bolsters the case for dynamicism.

In the literature, Brown (2005) famously argues for dynamicism, claiming that the genuine explanatory order runs from the dynamic laws to chronogeometric laws. However, his arguments seem to consist largely of rhetoric and therefore not very clear or developed. For example, he compares the explanatory power of spacetime and its geometry to the role played by “the geometries of the configuration space in classical mechanics” and “the space of equilibrium states in thermodynamics” (139), which presumably do not play an explanatory role. He does not go further into justifying the comparison but stops with the question “why should spacetime geometry be any different?” (139) He also argues that “the conspiracy of inertia” (141) is a postulate without no explanation, and “anyone who is not amazed by this conspiracy has not understood it” (15). He goes on to claim that, with dynamicism, inertial motion is finally not “a miracle” (163) Understandably, geometricists feel unengaged as a result, who hold the geodesic principle as fundamental and in no need of further explanation. For example, Martinez (2007) complains: “inertial motions are distinct, having different speeds and directions; how much more different would they have to be to not constitute a conspiracy?” (211)

While I think Brown’s rhetoric does succeed in prompting us to reconsider the intuitive explanatory order from spacetime geometry to dynamic laws, for further progress, we must go beyond merely wrestling with our intuitions and give non-question-begging reasons for favoring one explanatory order over the other. I aim to offer some such reasons: geometricism not only postulates a more complicated ontology and ideology, which leads to more problems of empirical underdetermination, but also imposes unnecessary constraints on the behaviors of matter.

Another problem with Brown’s advocacy for dynamicism is that he does not distinguish between Weak (spacetime manifold exists) and Strong (spacetime manifold
does not exist). I will return to this problem in the next section when I discuss some objections to dynamicism. Here, I will engage with both versions of dynamicism, making it clear that Strong has advantages over Weak.

**Simplicity** Comparing to geometricism, dynamicism posits fewer kinds of entities and fundamental structures of the world. In particular, Strong posits fewer kinds of entities because it does not posit substantive spacetime, which is a very different kind of object from matter.\(^{11}\) (As Newton put it, space is “neither substance nor accident” but “has its own manner of existence” (De Grav).) Dynamicism posits fewer fundamental structures because it does not posit chronogeometric structures as basic, such as spatiotemporal intervals featured by geometricism.

While there is often a trade-off between ontological parsimony and ideological parsimony, it is not clear at all that dynamicism needs to posit more axioms than geometricism, if not the opposite. For example, in special relativity, dynamicism only has one basic principle constraining all dynamic laws, while geometricism has a lot more. One may object that this is not a matter of simple counting—perhaps the second-order form of Dynamicism\(_{STR}\) is inherently more complex despite it being one principle. But the second-orderness is not essential here, since the principle can just as well be about the coupling of the gravitational field and other matter fields, as shown in Dynamicism-1.\(^{12}\)

**Empirical underdetermination** When one theory has more fundamental structure than another without empirical difference, it can be expected that the former would be inflicted with more problems of empirical underdetermination.

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\(^{11}\) The number of entities are similar, since for dynamicism, the gravitational field is a fundamental entity (in the context of relativity) in the place of spacetime for geometricism. But the gravitational field is not a different kind from other matter fields.

\(^{12}\) I would also like to make a connection between dynamicism and fundamental “invariance groups” entertained by structuralists (for example, see McKenzie 2013, French and Ladyman 2003). The invariance groups consist of symmetries of physical fields, so the approach amounts to taking the symmetries of physical fields as fundamental, which is very much in the spirit of dynamicism.
Consider two famous arguments against realism about spacetime and geometry from the literature. The argument from the Poincare disk says that the citizens of a peculiar two-dimensional disk space cannot distinguish between two possibilities with any empirical measurements: a hyperbolic geometry with no universal force and a Euclidean geometry with a universal force (that distorts measurement devices). I do not discuss how problematic this problem is for geometricists, but for those who are bothered by this argument, dynamicism provides a principled solution. According to dynamicism, there is no separate metric field that determines the real geometry, the measurements of which can be distorted by a universal force. There are only dynamic laws that are fundamental and determine the apparent geometry.

The hole argument says that if spacetime is substantive, the world would be radically indeterministic because fixing all physical states outside a region (the “hole” in “the hole argument”) does not fix the physical states inside it no matter how small the region is. This is a problem since our world should not be “fated” to be indeterministic as a consequence of metaphysics. There is, of course, an extensive literature on how to respond to the argument on behalf of substantivalism (e.g., Butterfield 1989, Hoefer 1996, Pooley 2006, Weatherall 2008), and I am personally sympathetic to many of the responses. Nonetheless, as far as the hole argument remains a force against substantivalism for some people, it helps motivating the rejection of substantive spacetime. Dynamicists can answer the argument by denying the reality of manifold and spacetime points (see Brown 2005, 156). The apparently different physical situations in the hole arguments only amount to using different coordinate systems—a mere renaming of the relative positions of the metric field and other matter fields—and therefore are not genuinely different. Note that this response is only available to STRONG that denies the existence of manifolds.\(^\text{13}\)

\(^{13}\)Indeed, if spacetime manifold is posited, the dynamicists would have fewer resources to solve the hole argument. For example, metric essentialism—the view that metric is essential to spacetime points—which is proposed by some geometricists (e.g., Maudlin 1988) as a response to the hole argument is not available to the dynamicists.
Unwarranted restrictions  The null hypothesis says that light ray travels along null geodesics in vacuum. Some authored such as Menon et al (2018) have pointed out that the null hypothesis is not derivable from the Einstein-Maxwell equations (relativistic electromagnetism). As we known, light is an electromagnetic wave and the ray-like trajectory of light is not always well-defined. For example, in optics, Poisson spot is the famous phenomenon that there will be a bright spot in the middle of the shadow when light passes around a smooth circular object. But light can obtain a ray characterization through geometric optics approximation, which holds when the wavelength of the light in question is significantly smaller than (1) the length scale on which its amplitude varies, and (2) the scale on which the curvature of spacetime is nonnegligible. The approximation can fail precisely in cases like Poisson spot. The null hypothesis can be obtained as the limit of geometric optics, where the wavelength of the light tends to zero while its amplitude remains constant. But as Menon et al point out, this derivation is based on the assumption that the solutions to the Maxwell equations arrived at in this limit do approximate exact solutions to arbitrary accuracy. This assumption is nontrivial: it can be violated in some physically and epistemically relevant scenarios.

There is an extensive literature on the derivation of the geodesic principle from the Einstein field equations (e.g., Misner, Thorne and Wheeler 1973), in which it is often claimed that the geodesic behavior of a free falling system is a result of the vanishing covariant divergence of its mass-energy tensor, which in turn follows from the Einstein field equations (see Brown 2005, Malamant 2012, Weatherall 2011, Menon et al 2018). It is usually considered an achievement of general relativity to prove the principle rather than postulating it. However, various authors such as Tamir (2012) and Weatherall (2011) have pointed out that this derivation actually relies on additional assumptions. According to Tamir, we can only reasonably derive that free falling systems tend to cluster around time-like geodesics without achieving
exact “geodicity.”

The derivation of the clock hypothesis relies on the validity of local Lorentz invariance. Recall that in special relativity, this principle is derived from Lorentz invariance of physical laws that govern the matter constitutions of clocks. If Lorentz invariance holds of small areas where gravitational tidal effect is small, then we can recover the principle locally. And if clock surveys time-like intervals of its worldline locally, then adding them together gives the length of its worldline, which recovers the principle globally. However, local Lorentz invariance is not a consequence of the field equations. If we interpret “local” as infinitesimally small, then this amounts to the requirement that the gravitational field has Lorentz signature \((-+++)\). (In geometricism, this requirement is built into Geometricism-2, namely how the spatiotemporal intervals are locally determined by the metric.) While the Einstein field equations typically apply to Lorentzian metrics, they can also apply to other metrics (see Dray et al 1996 for a variational metric as a solution to the field equations). Thus, I am not convinced that the clock hypothesis is necessarily true in the context of general relativity.

According to these results, the chronogeometric-dynamic laws postulated by geometricism are only approximately true, which is a tell sign that they are derived from more fundamental principles. Positing them as basic principles that constrain the behavior of matter is unwarranted because these additional constraints not only do not account for the empirical success of general relativity but may even be falsified by further empirical data. In contrast, dynamicism does not need to posit these principles in addition to the Einstein field equations (and principles of matter theories), and therefore does not impose these unnecessary constraints on the behavior of matter.

**The pathway to quantum gravity** Finally, I would like to comment on how dynamicism fares better than geometricism when we go beyond general relativity, in particular, concerning quantum theories. The intuition behind attributing basic
chronogeometric significance to the gravitational field has a lot to do with the special status of the field, namely that all matter fields are coupled with it. However, when we consider the programs of quantum gravity, the gravitational field shares overwhelming similarities with other matter fields. It not only carries energy and momentum like other matter fields, it is also subject to quantum fluctuations.\(^\text{14}\) Although we do not yet have a fully formulated theory of quantum gravity as a candidate fundamental theory, there is however a fully formulated and mathematically consistent “effective” theory of gravity which holds at physical scales that significantly larger than the Planck scale (but can still be smaller than any scale that we can probe with our current technology). The prescription for formulating this effective theory of gravity is exactly the same as formulating other effective theories of particle fields. For example, in this framework, a “graviton” is an excitation of the gravitational field (perhaps we should call it the graviton field now) in exactly the same formal sense as a photon being an excitation of an electromagnetic field in quantum electrodynamics. (For more details of the effective theory of gravity, see Burgess 2003, Donoghue 2012, Crowther 2013, 2016, [Anonymized])

Thus it seems that we have good reasons to consider the gravitational field on a par with other matter fields. The geometricists, then, face a dilemma. It is implausible to insist that the gravitational field is just a mathematical device for encoding the chronogeometric properties of spacetime.\(^\text{15}\) But if they posit that the field is a real physical entity that lives on spacetime and is governed by dynamic laws (or effective dynamic laws in aforementioned effective theory) just like other matter fields, then it would be odd to consider it also to have the fundamental chronogeometric significance

\(^{14}\)It is widely accepted that the metric field is subject to quantum effect (e.g., Eppley and Hannah 1977). Semi-classical gravity with a classical metric field is treated as only an approximation to the fundamental theory, although there are some skeptical voices (e.g., Mattingly 1999).

\(^{15}\)It is worth mentioning that although many programs of quantum gravity are labeled as quantum geometry, they are not really committed to geometricism about the gravitational field. For example, Rovelli and Vidotto (2015) argue that spacetime geometry is discrete and fuzzy. Such a statement merely rests on the common assumption that the gravitational field permits a geometric interpretation. Indeed, Rovelli seems to be against geometricism (see Rovelli 1997, Brown 2005).
on top of those dynamic laws.

I should briefly note that many approaches to quantum gravity struggle with the problem of background dependence: they have to posit an undesirable fixed background spacetime (i.e., not subject to quantum effect) in addition to the quantized gravitational (or graviton) field. But this is not a vindication of geometricism. On the contrary, since background dependence is seen as a clear problem to be solved, dynamicism plays a good methodological role in putting extra pressure on solving this problem, or further clarifying the problem.

3 Revisiting Two Objections to Dynamicism

Norton (2008) raises what I consider to be two important and perhaps representative objections to dynamicism. In essence, he argues that dynamicism leaves the coincidence among different matter theories unexplained, and that dynamicism tacitly assumes spacetime as a fundamental entity. The first objection applies to both version of dynamicism (Weak and Strong), and the second only targets Strong and does not have force towards Weak. But this objection should still be taken seriously if we want to keep all the important virtues of dynamicism that are available only to Strong (Section 2.3). To fully respond to this objection, I content that we need to go beyond the traditional framework of differential geometry as well as other existent proposals (in particular, Menon’s (2019) proposal that I will discuss in the next section).

First, Norton argues that dynamicism has to either (1) leave a coincidence among different matter theories unexplained, or else (2) commit to an extreme form of operationalism. For example, clocks tend to synchronize regardless of their particular constitutions, which are studied by different matter theories. If there is no such thing as time or time-like intervals, this would seem to be a mysterious coincidence. But
there is no such mystery: they synchronize simply because they track the metric of spacetime in accordance to the clock hypothesis. Similarly, why do fast moving rods appear shorter regardless of their particular matter constitutions? Again, this is not a mysterious coincidence and the reason is that they are correctly tracking the spacetime geometry responsible for the phenomenon of length contraction. Norton compares this explanatory task to “footprints on a sandbeach”: if we see many identical footprints on a sandbeach, we will infer that there was a foot that produced them. Thus we should infer that spacetime exists and that rods and clocks measure the metric of spacetime. But then, one has to accept a spatiotemporal structure existing prior to matter, unless one embraces extreme operationalism, according to which quantities like temporal durations do not exist until they are measured.

Since the second horn of the dilemma is indeed implausible, the core of the argument lies in the first horn. In special relativity, does the fact that all physical laws are Lorentz invariant imply that there is a Minkowski spacetime? Not necessarily. The principle of common cause that coincidences should be explained by a common cause is really grounded in that the resulting theory that posit the common cause is simpler or has other theoretical virtues—if this were a fundamental methodological principle, we would be pursuing an unreasonable quest for explaining every principle (e.g., see the discussion in van Fraassen 1981). But as we see in Section 2, it is not clear at all that the geometric interpretation of special and general relativity is simpler than the dynamic one, if not the opposite. The common symmetries of different matter theories is not analogous with the case of footprints on a sandbeach. In the case of footprints, the theory that posits a foot is likely to be much simpler than a theory that does not (what causes the identical footprints if not a foot?) Note that dynamicists can also explain the common symmetries by fact that all matter fields are coupled with the gravitational field, and again the resulting theory has no less theoretic virtues than the geometric interpretation. Thus I think dynamicists
can respond to Norton’s first objection by simply rejecting the first horn of Norton’s argument.

Norton’s second objection is that dynamicism presumes spacetime through its use of coordinate systems. Indeed, the use of coordinate systems is pervasive in dynamicism. For instance, physical quantities and laws are Lorentz invariant just in case their expressions have the same form in a set of coordinate systems related by Lorentz transformation. Norton argues that the use of such coordinate systems is problematic because it presumes spacetime coincidence. For example, consider two clocks of different material constituents and governed by different matter theories recording the same period of time while moving away from each other. We marked one as traveling from parameter \((0,0,0,0)\) to \((x,0,0,t)\) in one matter theory, and another as traveling from \((0,0,0,0)\) to \((-x,0,0,t)\) in the other matter theory. But to assume that the coordinate parameter \((0,0,0,0)\) in both matter theories refers to the same thing is to introduce spatiotemporal coincidence, which tacitly assumes an underlying spacetime.

Norton preempts two immediate responses. First, he argues that spatiotemporal coincidence cannot be inferred from the coupling—that is, the interaction—of different matter fields. The spins of two particles can be coupled without indicating that they spatiotemporally coincide—it just indicates the correlation of their spin momenta. Second, he rejects the response that we should formulate all matter theories in one set of parameters, in which case a parameter like \((0,0,0,0)\) automatically refers to one and the same thing. But we cannot simply assume that all matter theories can share a set of parameters without assuming spacetime—after all, we do not assume that all particles share the same internal spin space.

Can we justify the presumption of spatiotemporal coincidence through the universal coupling of matter fields with the gravitational field? I do not think so, because in specifying a particular coupling, i.e., how the value of a field correlate to the value
of another field at certain points, we already assume that the gravitational field and other matter fields can spread over the same coordinate system (or the same manifold), which is exactly what Norton challenges.

It is worth clarifying again that dynamicists do not necessarily reject a substantive spacetime.\textsuperscript{16} According to Weak, there is a metrically amorphous manifold that is our spacetime. It follows that Geometricism-1 is true. So those dynamicists who accept Weak would not reject Geometricism-1 but only the other two principles of geometricism. But to follow this strategy, we have to give up Strong and its important benefits. If there is a real manifold underlying matter fields (including the gravitational field), there would be the question whether a spacetime point $P$ is occupied by (say) my laptop or not, which is empirically inaccessible. In that case, the dynamicists would lose their distinct resource to solve the hole argument comparing to the geometricists.

To take stock, Norton objects that if the dynamicists want to do away with manifolds, the use of coordinate systems still tacitly assumes a substantive spacetime. In other words, not all primitive geometric notions (broadly construed) can be explained away by dynamicists—the notion of spacetime points required for spatiotemporal coincidence is still presumed. This, I think, is an important worry against Strong, and I submit that we must appeal to a different framework that neither posits a manifold nor depends on a coordinate space.

\textsuperscript{16}As I mention in Footnote 1, Brown’s version of dynamicism is ambiguous. To remind: Norton reports Brown’s feedback to his argument:

Harvey Brown has assured me that Brown and Pooley ([2004]) and Brown ([2005], especially Ch. 2) presumed the existence of a manifold of spacetime events with coordinate systems... these events and coordinates are posited by Brown and Pooley’s texts as spatiotemporal structures, independent of matter, as opposed to ones that can arise only in the presence of matter. Specifically, the analysis shows that they have no choice but to make the posit; attempts to extend their constructivism to the manifold of spacetime events will fail. (11-2)

Here, Brown is claimed to endorse the reality of spacetime manifold existing prior to matter. But in the same book, Brown also explicitly denounces spacetime manifold in reply to the hole argument (156).
4 Algebraic Dynamicism

In the standard framework, the coupling (or interaction) between two fields is expressed through defining them on a common spacetime, manifold or coordinate system. This leads to the objection that whatever serves the role of this common background is spacetime, and therefore dynamicists cannot consistently hold that spacetime is not part of fundamental reality. To respond to this challenge, it is the best to appeal to a framework that does not construe fields as living on a manifold or a coordinate system so that it is clear that spacetime coincidence, along with other chronogeometric structures, can be derived from rather than being presumed by dynamic laws. The best candidate for such a framework is perhaps algebraicism, in which fields are elements of algebraic structures without an underlying manifold. Indeed, Menon (2019) proposed a response to Norton based on this approach. However, I will argue that a solution based on traditional algebraic formalism (such as Menon’s) faces serious problems. Thus, we must go beyond this formalism. I introduce the technical framework advanced by Chen and Fritz (2021) and discuss how it can help solve the problems.

It is worth highlighting that the formalism of algebraicism wasn’t proposed for the purpose of dynamicism. Geroch proposed Einstein algebras in the hope of accommodating quantum theory, which calls for a “smearing-out” of spacetime points. Later, Earman (1989) suggested using algebraicism as a response to the hole argument, though apparently not very successfully. Algebraicism also provides potentially more flexible framework for physics. For example, we could have algebraic structures that do not correspond to any geometric structures with points, such as the noncommutative algebras on which noncommutative geometry builds (see Doplicher,

\footnote{Rynasiewicz (1992) pointed out that algebraicism does not “smear out” spacetime points because points can be reconstructed from the algebraic structure, and it does not avoid the hole argument because the indeterminism among isomorphic algebras is analogous to that among diffeomorphic manifolds. Bain (2003) disputes this claim.}
Fredenhagen, and Roberts, 1995; Marcolli, 2018; Huggett, Lizzi and Menon, 2021). It may be helpful in formalizing quantum theories, e.g., see Bain (2003) and Heller and Sasin (1999). There is also an added benefit of having a natural account of tangent spaces in this framework ([Anonymized]).

4.1 The Traditional Algebraicism and Its Problems

More elaborately, general relativity is standardly formulated in the framework of differential geometry, which features manifolds with topological and differential structures. Algebraicism provides an alternative framework for general relativity that does not posit manifolds. Consider a 4-dimensional smooth manifold \( M \). Now, consider all the real-valued smooth functions on \( M \). We can operate on these functions in various ways. For example, we can add or multiply two smooth functions to get another smooth function. Now, instead of considering these smooth functions as maps of \( M \to \mathbb{R} \), we can instead consider them abstractly as elements of an algebraic structure \( C^\infty(M) \) defined by algebraic operations such as addition and multiplication (for more details, see Geroch 1972 and Moerdijk and Reyes 1991). This algebraic structure is the basic entity postulated by algebraicism. The idea behind algebraicism is that all the relevant information about manifolds can be encoded in these algebras of smooth functions (see Connes 2013). In particular, all geometric entities that we need to do physics up to general relativity, such as vectors and tensors, can be defined in algebraic terms without any reference to manifolds. In this formalism, the manifold-theoretic structure \( \langle M, g \rangle \) is replaced by \( \langle C^\infty(M), \hat{g} \rangle \) (called “Einstein algebra”) where \( \hat{g} \) is an algebraic equivalent of the metric field, as solutions to the Einstein field equations (see Rosenstock et al. 2015).

We can interpret \( C^\infty(M) \) realistically as consisting of all possible configurations of a scalar field (see Earman 1989; Demaret, Heller, and Sasin, 1997; Bain, 2003). Again, a scalar field configuration here is not a map from manifold points to real numbers,
but a simple element in the basic algebraic structure. Under this formalism, we can derive the notion of spacetime coincidence without tacitly assuming spacetime at the fundamental level. For example, consider elements $\Phi, \Psi$ of $C^\infty(M)$ which can be respectively realized as a function with value $n$ at point $p$ (and negligible elsewhere) and a function with value $m$ at $p$ (and negligible elsewhere). This spacetime coincidence of $\Phi, \Psi$ at $p$ is already encoded in the algebraic structure. For example, let $\Gamma$ be the element that can be realized as having value $m + n$ at $p$ (and negligible elsewhere), and $\Xi$ be the one realizable as having $n$ at $p$ and $m$ at $q$ (and negligible elsewhere). The spacetime coincidence of the peaks of $\Phi, \Psi$ is encoded in the fact that the addition of $\Phi$ and $\Psi$ is $\Gamma$ rather than $\Xi$. Menon (2019) proposed a response to Norton based on this approach.

However, this formalism faces serious problems as a solution to Norton’s argument against Strong. Recall that two important motivations for Strong against Weak are that the former is more ontologically and ideologically parsimonious, and that only the former provides a clean response to the hole argument (Section 2.3). However, both virtues will be violated by the current algebraic formalism—thus this solution is a mismarriage between algebraicism and dynamicism. Let me explain.

According to this approach, there is no spacetime manifold, but there is an algebraic structure $C^\infty(M)$ at the fundamental level. What is this structure? We can read it realistically as consisting of configurations of a real-valued scalar field. But what is this scalar field? There are no physical scalar fields at the fundamental level acknowledged by current physics—the closest is the Higgs field (a complex-squared-valued scalar field). But even if there is actually a fundamental scalar field, there are two problems. First, this formalism would be privileging one physical field ontologically. For example, following the prescription that all relevant structures are to be defined in terms of the algebra $C^\infty(M)$, non-scalar fields such as the electromagnetic field then would have to be defined in terms of the scalar field, but it is implausible
to think that the electromagnetic field is ontologically dependent on the scalar field. Secondly, even if there is an actual scalar field, it’s possible that it does not exist, but according to the current formalism, such a field has to exist. Moreover, if $C^\infty(M)$ is interpreted as a physical field not acknowledged by current physics, then we would be positing a fundamental “ghost” field that play no role in our best physical theory. Now, this would especially trouble dynamicists rather than geometricists or substantivalists who adopt the algebraic approach. From the latter’s perspective, this field is simply the algebraic counterpart of spacetime. (Since the natural interpretation of the algebraic structure is substantivalist, this approach can be called “substantival algebraicism.”) See Bain 2003, Earman 1989) But this is not a good option for the dynamicists. Just like spacetime, $C^\infty(M)$ would be a different kind of entity from all other physical fields—no ontological parsimony for dynamicists. The amount of fundamental structure is also equivalent to the manifold approach (Rosenstock et al 2015).

Next, the problem of empirical underdetermination, and in particular the hole argument, would reemerge in this formalism. In the manifold-theoretic case, the hole argument is premised on that two distinct models related by diffeomorphisms, such as $\langle M, g \rangle$ and $\langle M', g' \rangle$ where $g$ is carried to $g'$ by a diffeomorphism from $M$ to $M'$, can solve the Einstein equations (and field equations in general) equally well. Thus, there are distinct models that are empirically and dynamically indistinguishable, which is a problem. But now we can construct just as well two distinct models $\langle C^\infty(M), \hat{g} \rangle$ and $\langle C^\infty(M'), \hat{g}' \rangle$ that satisfy the algebraic Einstein equations. For concreteness, we can consider $M'$ as a proper subset of $M$ in both cases, and $g'$ is the restriction of $g$ to $M'$. Then $C^\infty(M')$ consists of elements of $C^\infty(M)$ restricted to $M'$ and $\hat{g}'$ is defined on $C^\infty(M')$ in the same way as how $\hat{g}$ is defined on $C^\infty(M)$. It follows that we

\footnote{Some people mean by “substantival algebraicism” that every element of the algebra is interpreted realistically. But here it means that the interpretation of the fundamental entities in the algebra is like spacetime.}
have distinct algebraic models (related by homomorphism) that are empirically and dynamically indistinguishable. This is just one more symptom of the problem that the algebraic structure $C^\infty(\mathcal{M})$ plays the same role as the spacetime manifold. Physical laws only specify how fields are defined on a manifold only up to diffeomorphism just as they specify how fields are defined on the basic algebraic structure up to homomorphism. Therefore, this formalism cannot help dynamicists answer the hole argument.

There is an added problem when we formulate the algebraic approach in the set-theoretic framework, recognized already by Earman (1989). Insofar as we have a principled problem of choosing between isomorphic manifold-theoretic models, we have a very analogous problem of choosing between isomorphic algebraic models. For example, we might think the algebraic model built on the set of smooth functions on $\mathbb{R}$ is distinct from the model built on the set of smooth functions on $(0, 1)$ simply because the two sets have different elements, even though it seems only reasonable to say that these are just two mathematical ways of talking about the same physical situation. We always have this problem in the set-theoretic framework regardless of how the structures are built upon the sets or how physical laws are formulated.

### 4.2 Dynamic algebraicism

To address these problems, we need a formalism of algebraicism that satisfies the following conditions to be happily wed to dynamicism: (1) it does not privilege a particular type of physical fields or posit a “ghost” field that plays the role of spacetime, providing structure for other physical fields. The ontology should consist of only fundamental fields recognized by physics. (2) The structure we impose on the physical fields should also be restricted to those that are required to do physics. This is to avoid the situation that, for example, two physical fields can be coupled in different ways that make no difference to physics. (3) We should adopt a framework
alternative to standard set theory so as to avoid the general problem of having distinct isomorphic models. I will argue that the framework proposed by Chen and Fritz (2021) formulated in category theory satisfies these conditions and is therefore a step in the right direction. I will call the solution based on this formalism dynamic algebraicism (a dynamically algebraic approach to dynamicism). The upcoming discussion will assume some minimal familiarity with basic category theory and therefore do not provide independent definitions of its technical terms.\textsuperscript{19}

To meet the first challenge, instead of positing $C^\infty(M)$ as the basic structure, we can let the basic algebraic structure consist of all physical fields recognized by current physics. But as we know, there are many types of physical fields with diverse mathematical representations: e.g., the electromagnetic field is a one-form, fermions fields are spinor fields, and the metric field is a tensor field. So how can we put them into one algebraic structure? Here’s the trick facilitated by category theory. First, we conceptualize a physical field as a field functor $F$ (equipped with a Lagrangian encoding its dynamic information). In pre-algebraic terms, a field functor is a functor from a customizable category of “spacetime” (in which the objects are certain representations of spacetime such as manifolds and the maps are certain transformations between them such as diffeomorphisms) to category of sets, assigning to every (say) manifold a set of field configurations, which commutes with diffeomorphisms between manifolds. The reason why we would want the assignments to commute with diffeomorphisms (that is, why fields should be conceptualized as functors) is that a physical field should be invariant under changes of coordinates (in the general sense that any transformations that do not make differences to physics are consider mere coordinate changes). To see the connection, let’s consider a simple example: consider a category that consists of all coordinate representations of Minkowski spacetime as its objects and all Lorentz transformations as its maps. A real-valued scalar field,

\textsuperscript{19}If the readers are not familiar with basic category theory, going through the wikipedia page on category theory would probably suffice for a general understanding of the discussion.
then, is a functor that specifies a smooth function for every coordinate representation of Minkowski spacetime that commutes with Lorentz transformation. This way, we build in the Lorentz invariance into the description of the field (note: the scalar field is not represented by a single smooth function but all of them related by Lorentz transformation).

To quickly forestall a common confusion, the appearance of manifolds or spacetime in the formulation of field functors is only provisional, and will be eliminated when the functors are reconceptualized as elements of the basic algebraic structure, as I will explain soon. This is analogous to the appearance of $\mathcal{M}$ in $C^\infty(\mathcal{M})$ under the traditional algebraicism.

Suppose we now have various field functors of interest including, for example, a scalar field, a tensor field, and a spinor field. How do we reconceptualize them (more precisely, their possible field configurations) as simple elements in a unified algebraic structure that does not include any other elements? In category theory, a natural transformation is a "higher-order functor" on the functors from one category to another that preserves the functor behaviors. Just as the smooth functions on manifolds can encode all information of manifolds in the traditional algebraicism, natural transformations between functors can potentially encode all information about the functors. Furthermore, we can define natural operators in terms of natural transformations (the only difference between them is that a natural transformation is a binary operator while a natural operator can be $n$-ary). Then we can reconceptualize the field functors as elements of the field algebra characterized by all the natural operators on the field functors. (For details on natural operators, see Kolar et al. 1993; for examples of field algebras, see Chen and Fritz 2021)

It is worth emphasizing that what natural operators we arrive at depends greatly on the details of the functors we use to represent various fields. There is a great deal of flexibility in customizing the functors in order to achieve various desired invariance
of fields in order to meet the second challenge. For example, an electromagnetic field (represented by a 1-form field) is invariant under gauge transformation—adding the gradient of a scalar field to the electromagnetic field does not correspond to any physical change (see for example Healey 2007 for a philosophical exposition). We should formulate functors in a way that identifies field configurations that are related by gauge transformation: instead of assigning all possible configurations of 1-form to a given manifold, we should assign only the quotient set of those configurations that does not distinguish between ones differing only by the gradient of a scalar field. This affects what natural operators we can define on the field.

Natural operators are abundant and some may not correspond to physical reality. To further meet the second challenge, the natural operators should be pared down to those that we need in our physical theories so that we do not have redundant structures in the field algebra that do not have a physical significance. For example, in order to formulate the Lagrangian of a scalar field $\phi$ according to a particular theory (such as $\phi^4$-theory), we need an operator on the scalar field that gives the “length” of the gradient of $\phi$, and this operator is one of the natural operators that characterize the field algebra, in which $\phi$ is an element.$^{20}$ But there are a vast number of natural operators on the field that are not required by physics.$^{21}$ What is the relevance of this? If we do not pare down natural operators to those that are required by physics, a homomorphism from one field algebra to another that preserves all physically-required operators but not all natural operators can generate distinct models that presumably do not correspond to genuine physical differences. Therefore, it is important that

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$^{20}$According to the massless scalar $\phi^4$-theory in $d$ dimensions—a commonly used toy example in physics—the Lagrangian density is given by $\mathcal{L}^{\text{scalar}}(\phi) = \left(\frac{1}{2} g^{\mu\nu} \left(\partial_\mu \phi \right) \left(\partial_\nu \phi \right) - \frac{\lambda}{4!} \phi^4\right) \sqrt{|\det g|}$. Here, the commutative binary operation $\langle d-, d- \rangle$ on $\phi \times \phi$ is a natural operation that gives us the “length” of the gradient of $\phi$.

$^{21}$For example, every smooth operator equipped by $C^\infty$-rings is a natural operator on a scalar field. (For every $n$-argument smooth function on $\mathbb{R}$, there is a corresponding smooth operator on a $C^\infty$-ring that takes its $n$ elements to another element. For more information on $C^\infty$-rings, see for example Moerdijk and Reyes 1991.) But what we need for physics up to general relativity is arguably only commutative rings, equipped with addition and multiplication (Geroch 1972).
we construct field algebras just with enough natural operators that are required for physics.

Finally, to meet the third condition, this approach is formulated within category theory, which does not distinguish between isomorphic structures. For example, recall that when we give set-theoretic constructions of a manifold, we can define a topological and differential structure on a set of spacetime points, or a set of singleton of points, or a set of real numbers—the resulting constructions would be distinct because set members are distinct. In contrast, the “internal” structures of sets that do not respect diffeomorphisms between manifolds are not expressible in the category of manifolds. Thus we cannot distinguish between diffeomorphic manifolds. Similarly, we cannot construct two distinct isomorphic field algebras within the same category of interest. (For more related discussions on category theory, see for example Awodey 1996, Bain 2013)

Let’s briefly go back to Norton’s objection. Since it has been shown in Chen and Fritz that this formalism is feasible for doing physics at least to a preliminary degree, and since that it posits no substantive spacetime or its substitutes, we can use it as an adequate response to Norton’s objection that dynamicism must presume substantive spacetime to account for spacetime coincidence. Unlike Menon’s solution, the elements in the field algebra are actual physical fields with energy and momentum and interactions. There is no tacit arena that physical fields live on, and the dynamic interactions of physical fields defined by natural operators are treated as primitive, from which spacetime coincidence and other geometric notions may arise.

Like standard dynamicism, there is no primitive chronogeometric significance of the gravitational field beyond the field equations. Note that if we restrict to special relativity, there is similarly no need for a separate gravitational field because the metric information can be encoded in the algebraic structure of the field algebra (Chen and Fritz 2021, 25-26). Furthermore, the hole argument does not apply be-
cause special care has been given to that the basic algebraic structures are defined through physically relevant operators and that we do not distinguish between isomorphic structures. Therefore, I submit that dynamic algebraicism is an attractive implementation of dynamicism (in particular, of STRONG) as well as an adequately dynamic implementation of algebraicism.

5 Conclusion

In this paper, I have motivated the dynamic approach to relativistic theories by highlighting how it addresses the shortcomings of the geometric approach. I also defend it against Norton’s objection by appealing to a new algebraic implementation of dynamicism called *dynamic algebraicism*, which I argue to be an improvement over both the standard version of dynamicism and the standard version of algebraicism.

According to this approach, there are no manifolds or any fundamental entity that plays the role of a substantive spacetime. Physical fields (and only physical fields) are fundamental, which are not defined on manifolds or coordinate systems but through the natural operators on them. Special attention is paid to positing only structures with physical significance, so we are in a better position to answer challenges from empirical underdetermination and radical indeterminism. In particular, since there is no substantive spacetime or its substitute, there is no objective reality about how metric field and matter fields spread over it, and the hole argument does not arise.

As a dynamic approach, the metric field in general relativity is treated as one of the matter fields that obeys dynamic laws. There are no spacetime geometry prior to the dynamical laws. In particular, the chronogeometric-dynamic principles are not posited as basic, but as approximate principles derivable from the field equations. Insofar as they cannot be derived, they are not taken to be true. This results in a more parsimonious ontology and a more flexible and empirically warranted theory.
References


